VECD (Visco-Elastic Continuum Damage): State-of-the-art technique to evaluate fatigue damage in asphalt pavements

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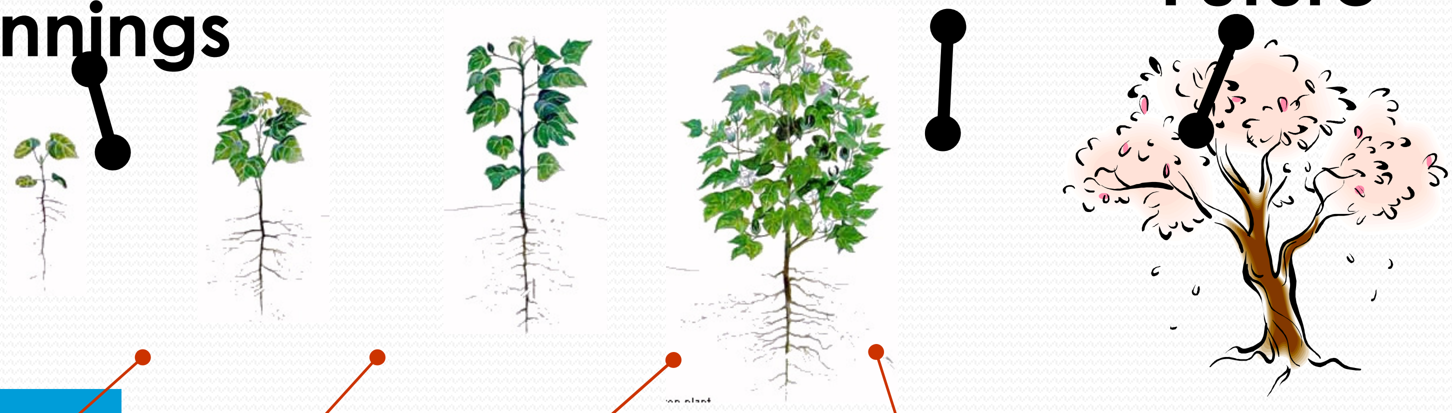
History of the Viscoelastic Continuum Damage (VECD) theory



Beginnings

Present

Future

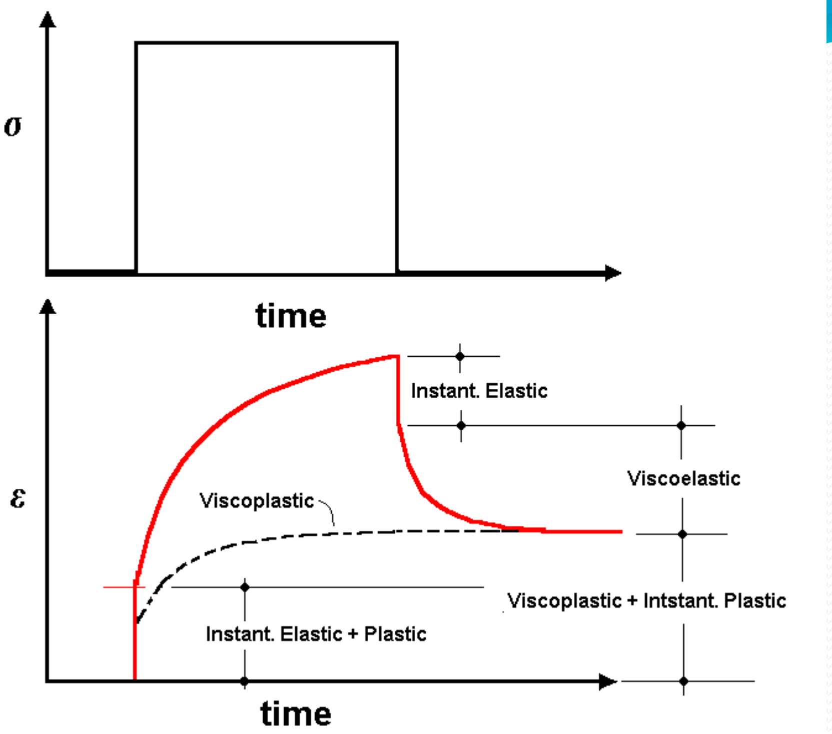
Schapery
U. Texas, Austin

Kim & Little
Texas A&M

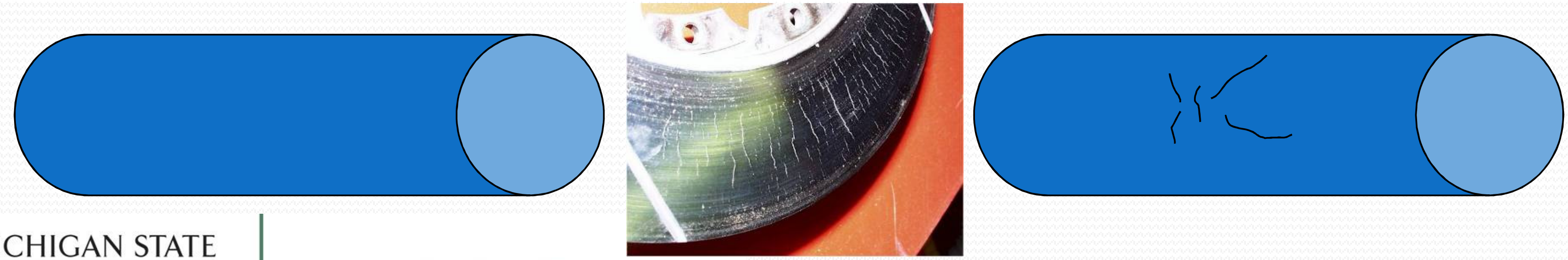
Kim, Daniel, &
Chehab
NCSU

Broader Research: U of Nebraska (Y.Rak Kim), Swedish Royal Inst. (Lunstrom & Isaacson), NCHRP 9-19 NCSU (Kim & Chehab) UMD (Schwartz & Gibson), AAT LLC. (Christensen & Bonaquist), FHWA (Kim et al, Kutay et al)

What is VECD?



- VE.. – Visco⅄elastic
    - Rate dependent
    - Fully recoverable
- ..CD – Continuum Damage
- A continuum is a body that can be continually sub-divided into infinitesimal small elements with properties being those of the bulk material.

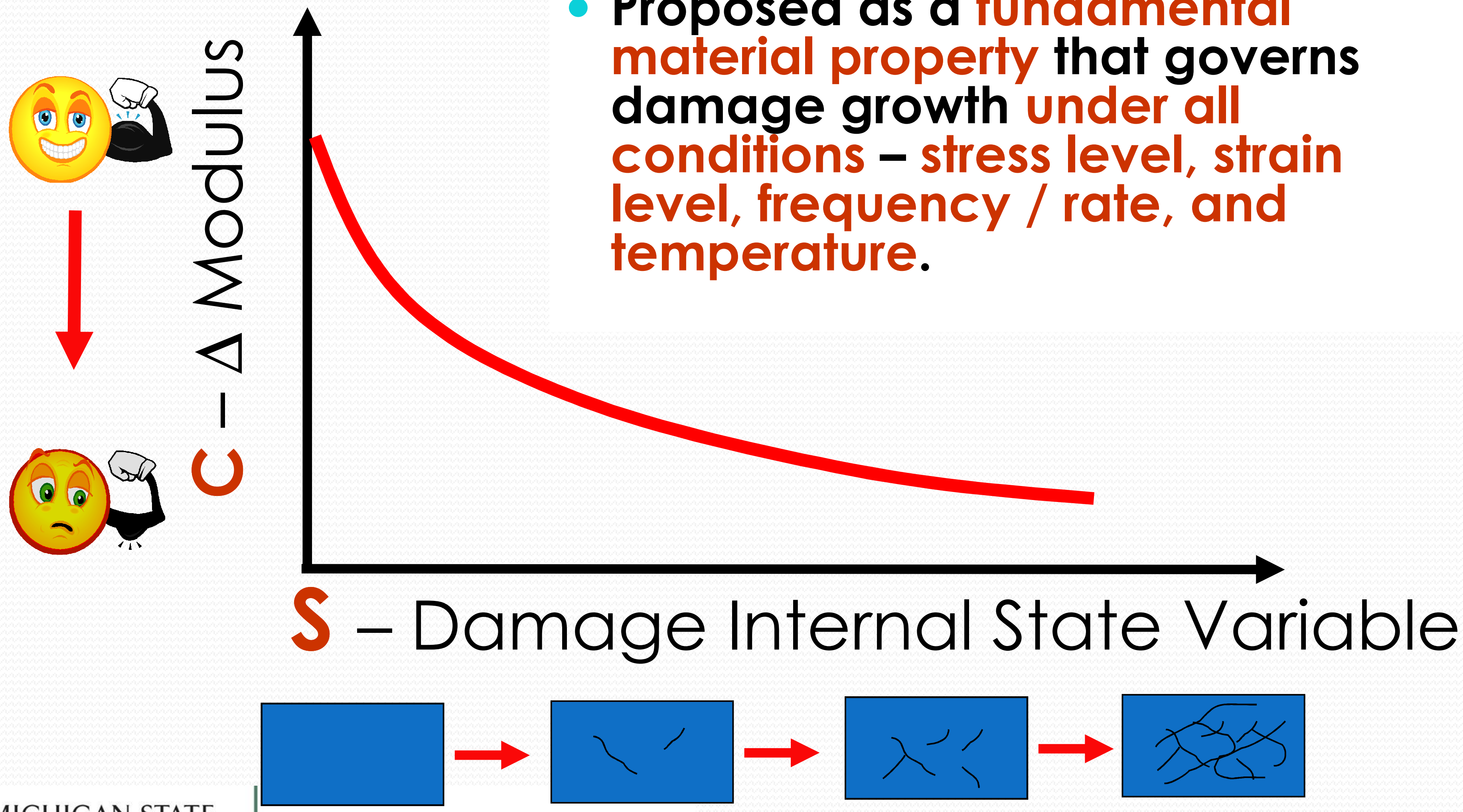


What is VECD?

	Elastic	**Viscoelastic**
No Damage	$\sigma = E\varepsilon$	$\sigma = E_R\varepsilon^R$ **E-VE Corresp. Principle** $\varepsilon^R = \frac{1}{E_R}\int_0^t E(t-t')\frac{\partial \varepsilon_{ve}}{\partial t'}dt'$ **Pseudo Strain**
With Continuum Damage	$\sigma = C(S)\varepsilon$	$\sigma = C(S)\varepsilon^R$

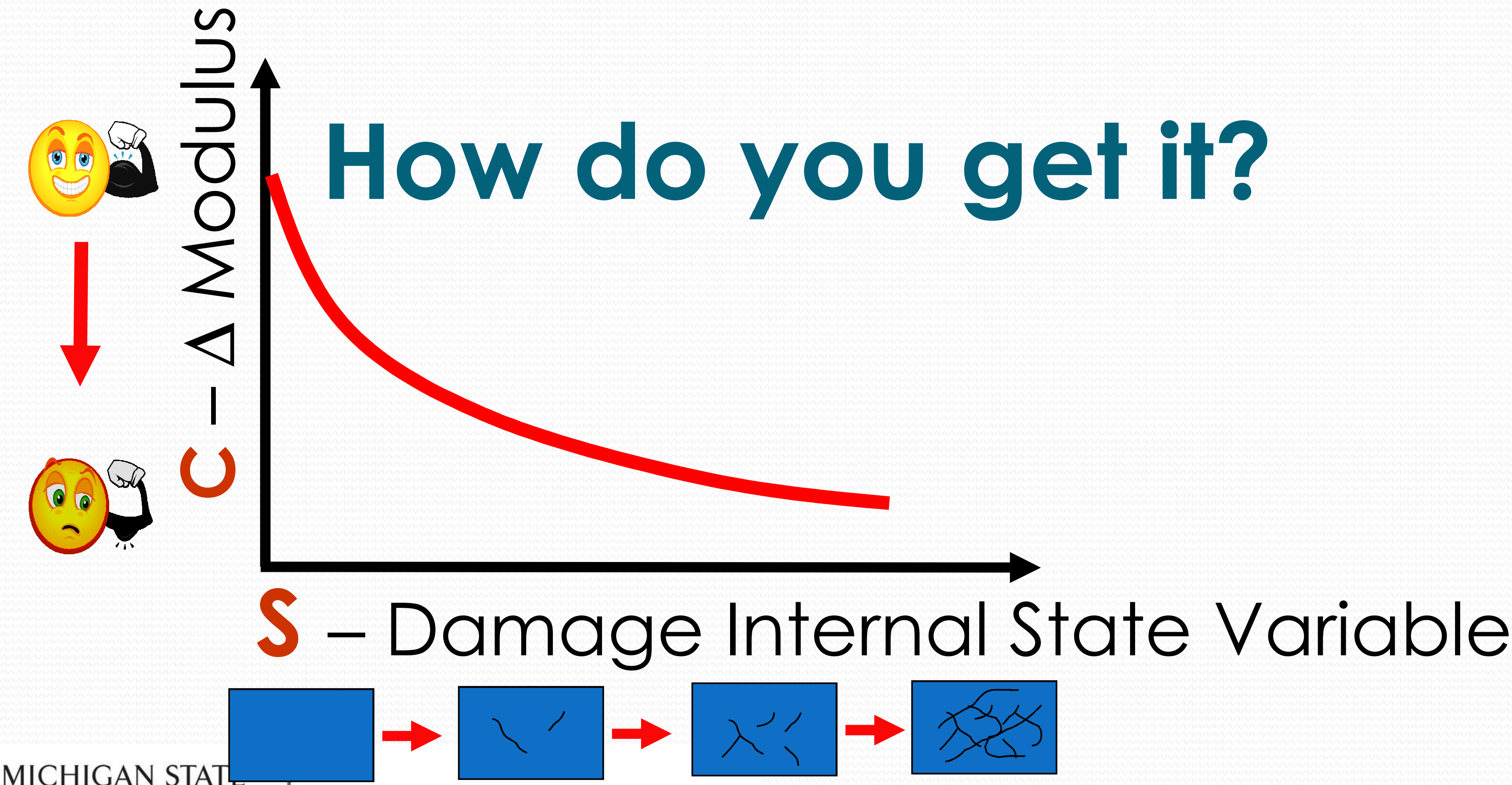
Damage characteristic curve (C vs S)

- **Proposed as a fundamental material property that governs damage growth under all conditions – stress level, strain level, frequency / rate, and temperature.**

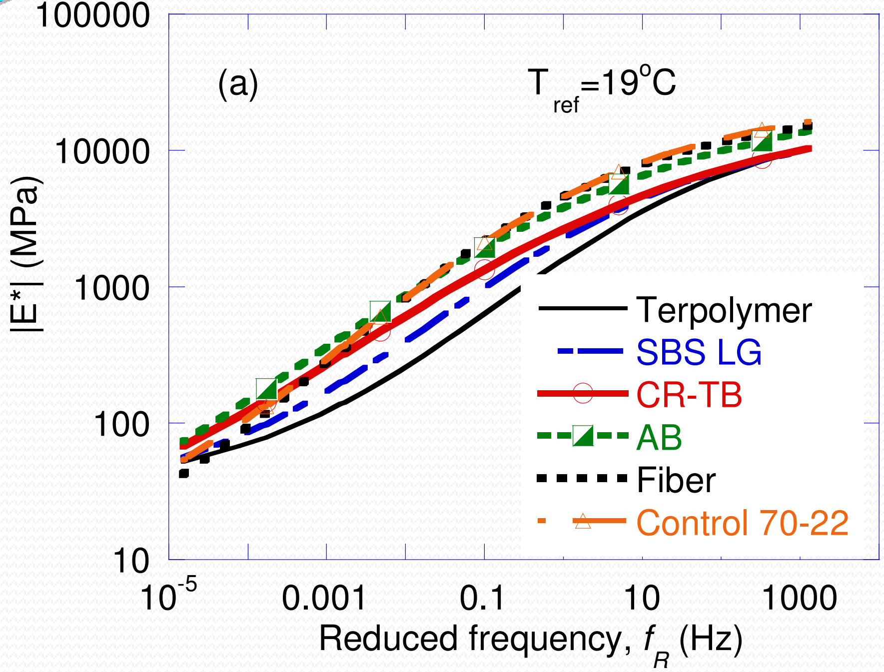


What can you do with C vs S curve?

How do you get it?

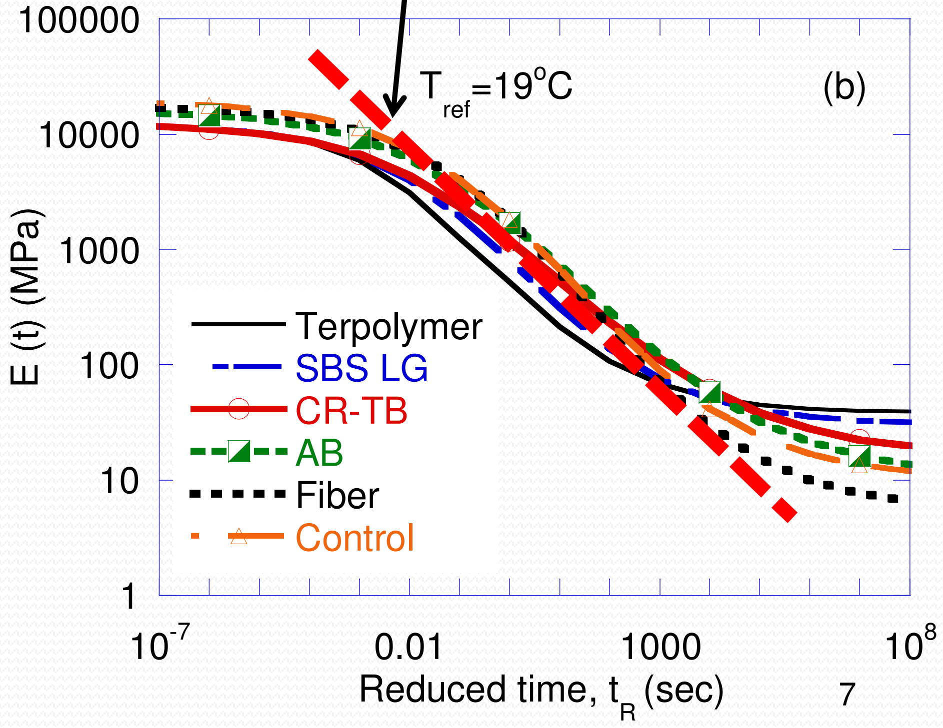


First input → LVE characteristics



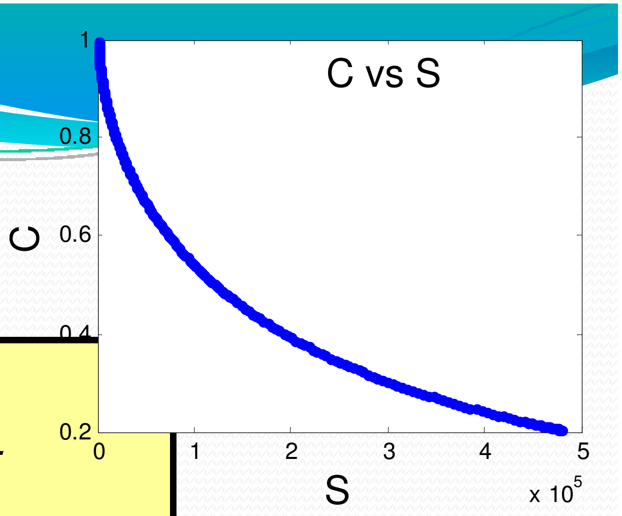
|E*| → E(t)
inter-conversion

Max slope = m,
α = 1/m



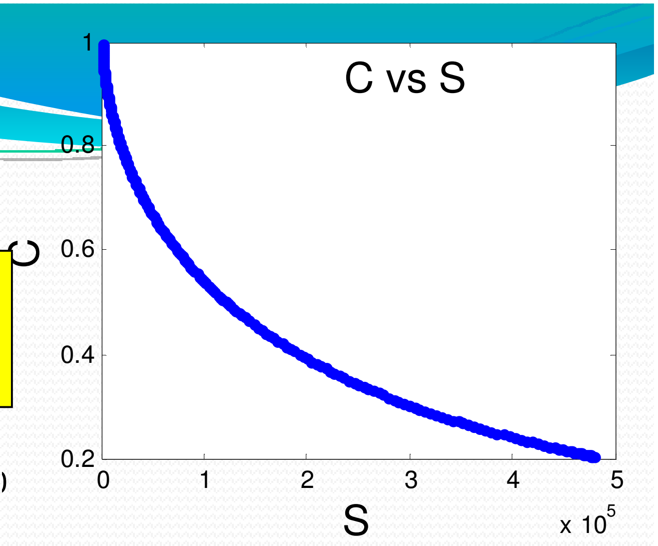
How to get C & S

VECD equations



Pseudo-strain	$\varepsilon^R = \frac{1}{E^R}\int E(t-\tau)\frac{\partial\varepsilon}{\partial\tau}d\tau$
Stress-strain relation (E-VE correspondence principle)	$\sigma = \mathrm{C}\ \varepsilon^{\mathrm{R}}$
Energy equation	$\sigma = \frac{\partial \mathrm{W}_\varepsilon}{\partial \varepsilon^{\mathrm{R}}}$ $\mathrm{W}_\varepsilon = \frac{1}{2}\mathrm{C}\,\varepsilon^{\mathrm{R}\,2}$
Damage evolution law	$\frac{d\mathrm{S}}{dt} = \left[-\frac{\partial \mathrm{W}}{\partial \mathrm{S}}\right]^{\alpha}$

How to get C & S



Input: $t,\ \boldsymbol{\sigma}(t),\ \boldsymbol{\varepsilon}(t)\ \ \&\ \ E(t)$

$$\varepsilon^R(t+\Delta t) = \int_0^{t+\Delta t} \mathrm{E}(t+\Delta t-\tau)\frac{\partial\varepsilon}{\partial\tau}\,d\tau$$

$$C(t+\Delta t) = \frac{\sigma(t+\Delta t)}{\varepsilon^R(t+\Delta t)}$$

$$S(t+\Delta t) = S(t) + \Delta t^{\frac{1}{1+\alpha}}\left[-0.5\ \varepsilon^R(t)^2\left(C(t+\Delta t)-C(t)\right)\right]^{\frac{\alpha}{1+\alpha}}$$

Drawbacks of past VECD approaches

- Monotonic tension likely requires force greater than capacity of the AMPT (a.k.a., SPT)

- Convolution integral can be very time consuming

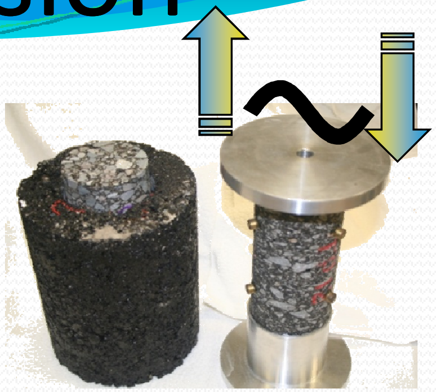
$$\varepsilon^{R}(t) = \int_{0}^{t} E(t-\tau)\frac{\partial \varepsilon}{\partial \tau} d\tau$$

- Routine FE analysis for pavement design may not be practical (but strong basis for Performance Related Specifications (PRS))

Practical use of VECD : Cyclic ‘Push-Pull’ Fatigue Approach

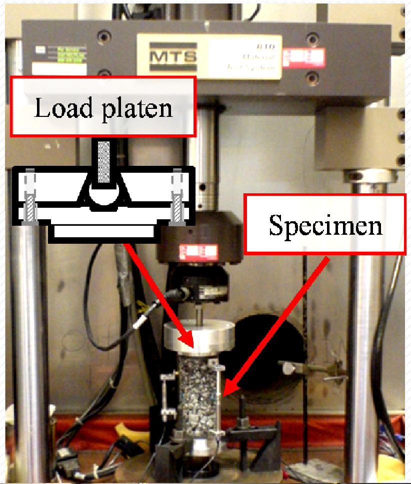
See Also: Kutay, Gibson &Youtcheff – AAPT – 2008

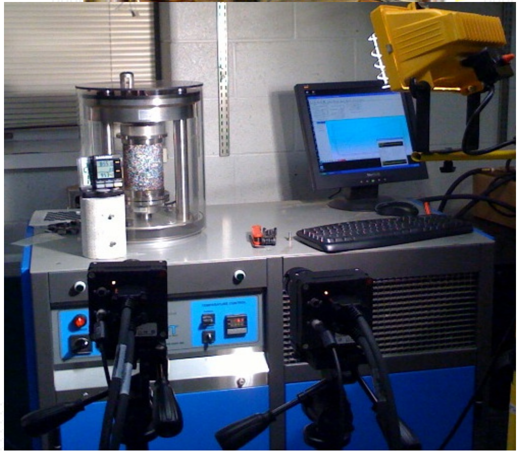
Uniaxial ‘push-pull = compression-tension’ fatigue tests



Advantages:

- Sample can be made in Superpave gyratory compactor
- Simple uniaxial stress state
- Tests can be conducted using the Asphalt Mixture Performance Tester (AMPT, a.k.a. Simple Performance Tester-SPT



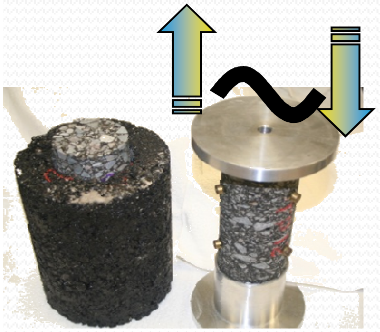


Cyclic 'Push-Pull' Fatigue Approach

- Well Poised For Implementation through AMPT (SPT)
  - **This type of routine testing is now within the reach of State DOTs and Contractors**

Currently Not in MEPDG

C & S from peak-to-peak stresses and strains



Input: $f_R, \left|E^*\right|_{LVE}, \alpha, \mathbf{N}, \boldsymbol{\sigma}_0^{\mathrm{N}}, \boldsymbol{\varepsilon}_0^{\mathrm{N}}$

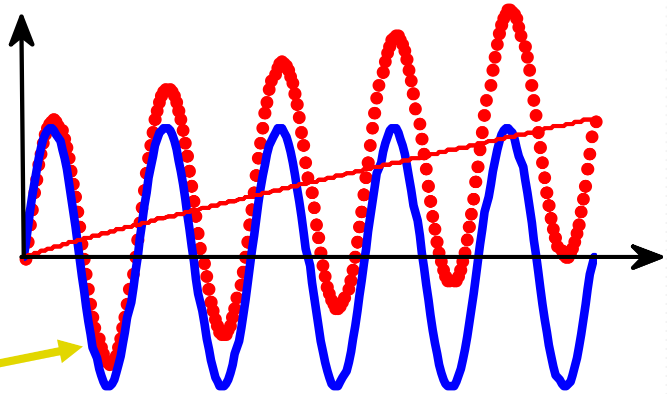
$$\left|E^*\right|_N = \frac{\sigma_0^N}{\varepsilon_0^N} \qquad C_N = \frac{|E^*|_N}{|E^*|_{LVE}}$$

$$\varepsilon_{\mathrm{N}}^{\mathrm{R}} = |E^*|_{\mathrm{LVE}}\ \varepsilon_0^{\mathrm{N}}$$

$$S_\varepsilon^{N+\Delta N} = S_\varepsilon^N + \left(\Delta N/f\right)^{\frac{1}{1+\alpha}}\left[-0.5\ \varepsilon_N^{R^2}\left(C_{N+\Delta N} - C_N\right)\right]^{\frac{\alpha}{1+\alpha}}$$

Derivations are at Kutay, Gibson & Youtcheff @ AAPT 2008

A simple Excel sheet is sufficient to obtain C & S

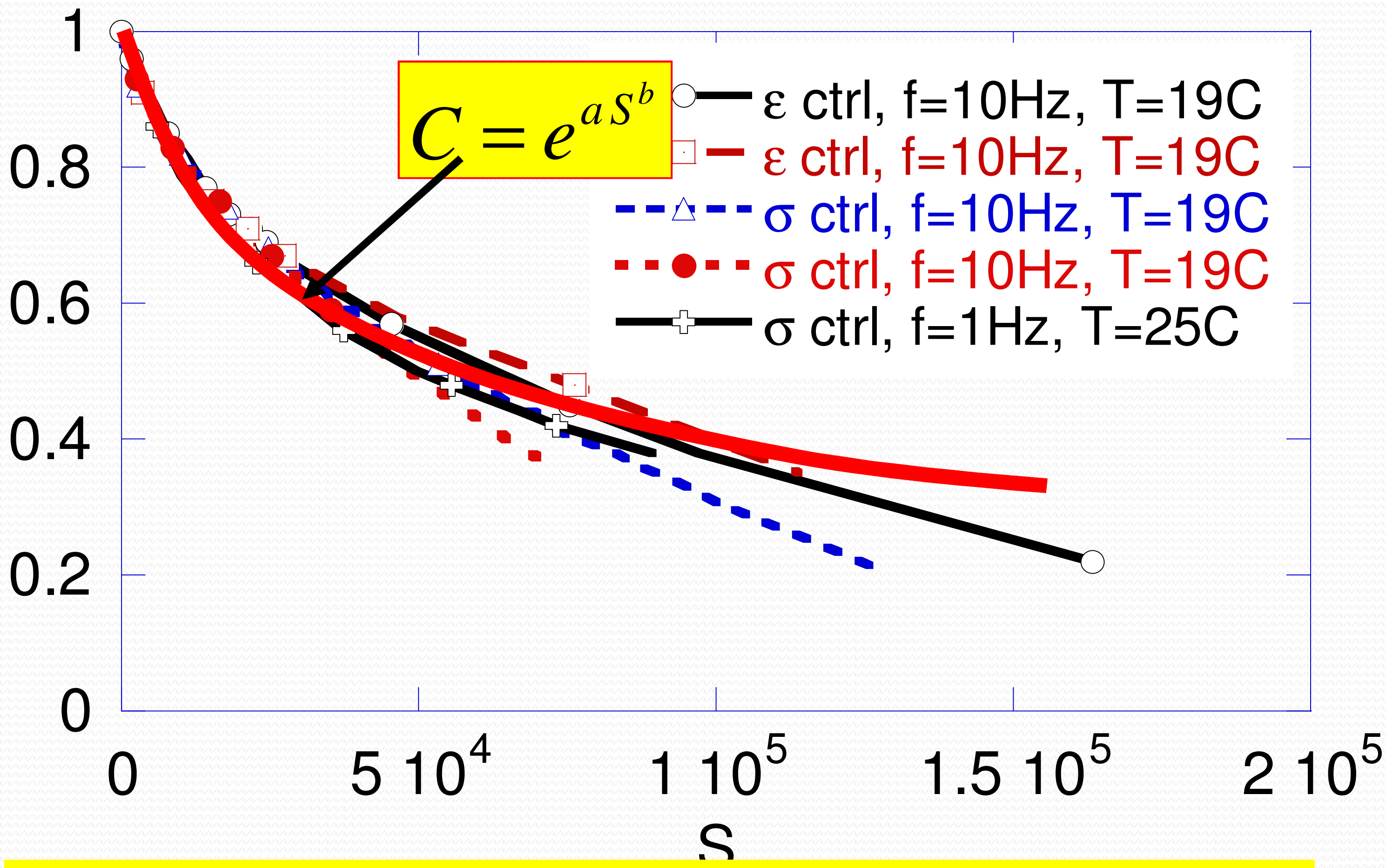


f (Hz)	10.0	$\lvert E^*\rvert_{LVE}$ (kPa)	6561000.00
T (^{o}C)	19.0	α (1/n)	2.4
Spec:	AB_SS1	I	0.7562878

N (Cycles)	σ^o (MPa)	ε^o_{avg}	$\lvert E^*\rvert$ (kPa)	ε^R (kPa)	σ^R (strain)	C	ΔN	ΔC	ΔS_ε	S_ε	Δ(1/C)	ΔS_σ	S_σ
1	0.193	3.88E-05	4962004.6	254.58	0.000029	1.00				0.00			0
2	0.248	5.00E-05	4961812.2	328.26	0.000038	1.00	1	0.0000	0.707	0.71	0.00	1.27E-10	1.27E-10
3	0.271	5.52E-05	4917405.8	361.98	0.000041	0.99	1	-0.0089	38.336	39.04	0.01	6.85E-09	6.98E-09
4	0.291	6.02E-05	4830195.2	395.19	0.000044	0.97	1	-0.0176	70.031	109.07	0.02	1.24E-08	1.94E-08
5	0.316	6.57E-05	4809764.2	431.21	0.000048	0.97	1	-0.0041	28.342	137.41	0.00	5.08E-09	2.45E-08
6	0.338	7.09E-05	4767497.1	465.33	0.000052	0.96	1	-0.0085	52.840	190.25	0.01	9.45E-09	3.39E-08
7	0.360	7.58E-05	4746788.1	497.50	0.000055	0.96	1	-0.0042	35.042	225.30	0.00	6.28E-09	4.02E-08
8	0.382	8.15E-05	4692761.3	534.73	0.000058	0.95	1	-0.0109	76.565	301.86	0.01	1.37E-08	5.39E-08
9	0.403	8.66E-05	4655022.2	568.49	0.000061	0.94	1	-0.0076	64.762	366.62	0.01	1.16E-08	6.55E-08
10	0.421	9.14E-05	4603999.2	599.73	0.000064	0.93	1	-0.0103	86.502	453.13	0.01	1.54E-08	8.09E-08
11	0.443	9.70E-05	4564073.4	636.59	0.000067	0.92	1	-0.0080	79.117	532.24	0.01	1.41E-08	9.51E-08
12	0.468	1.03E-04	4536579.7	677.24	0.000071	0.91	1	-0.0055	66.310	598.55	0.01	1.19E-08	1.07E-07
13	0.490	1.09E-04	4511095.7	712.16	0.000075	0.91	1	-0.0051	67.478	666.03	0.01	1.21E-08	1.19E-07
14	0.513	1.15E-04	4478648.6	751.57	0.000078	0.90	1	-0.0065	86.422	752.45	0.01	1.55E-08	1.35E-07
15	0.531	1.19E-04	4440024.7	783.95	0.000081	0.89	1	-0.0078	103.800	856.25	0.01	1.86E-08	1.53E-07
16	0.554	1.25E-04	4425322.2	821.23	0.000084	0.89	1	-0.0030	55.926	912.18	0.00	1.00E-08	1.63E-07
17	0.578	1.31E-04	4414818.2	859.21	0.000088	0.89	1	-0.0021	46.986	959.16	0.00	8.44E-09	1.72E-07
18	0.594	1.36E-04	4378763.3	889.44	0.000090	0.88	1	-0.0073	118.225	1077.39	0.01	2.11E-08	1.93E-07
19	0.616	1.42E-04	4352418.2	929.24	0.000094	0.88	1	-0.0053	100.718	1178.11	0.01	1.80E-08	2.11E-07

C vs S curve of the **Control PG70-22** mixture at different temp., freq. and loading modes

$$C = \frac{|E^*|_N}{|E^*|_{LVE}}$$



$$C = e^{aS^b}$$

$$S_{N+\Delta N} = S_N + (\Delta N/f)^{\frac{1}{1+\alpha}}\left[-0.5\, I\, \varepsilon_N^{R\,2}\left(C_{N+\Delta N} - C_N\right)\right]^{\frac{\alpha}{1+\alpha}}$$

Is peak to peak C vs S same as the C vs S computed using the hereditary integral ?

Validation methodology

STEP (1) Calibrate model using peak-to-peak formulation :

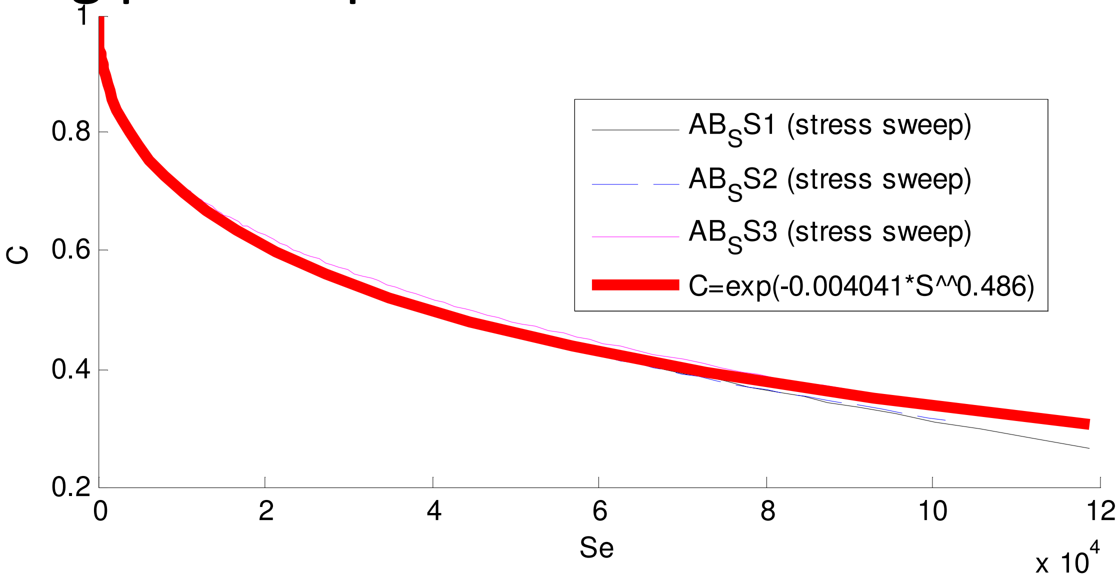
C vs S was calculated using peak to peak stresses and strains.

$$\varepsilon_N^R = |E^*|_{LVE} * \varepsilon_0^N$$

$$C_N = \frac{|E^*|_N}{I\,|E^*|_{LVE}}$$

$$S_{N+\Delta N} = S_N + (\Delta N/f)^{\frac{1}{1+\alpha}}\left[-0.5\, I\, {\varepsilon_N^R}^2\,(C_{N+\Delta N} - C_N)\right]^{\frac{\alpha}{1+\alpha}}$$

Fit $C = \exp(a\, S^b)$



Validation methodology

STEP (2) Simulate using the state-variable implementation of the hereditary integral: Rigorous simulation (input: time v.s. strain, output: stress) was performed using VECD state variable implementation.

$$\sigma_i^{el}(t) = e^{-\Delta t/\rho_i}\sigma_i^{el}(t-\Delta t) + \frac{\Delta\varepsilon}{\Delta t}\eta_i\left[1-e^{-\Delta t/\rho_i}\right]$$

(stress in each Maxwell element at time *t*)

$$\varepsilon^R(t) = E_\infty\varepsilon(t) + \sum_{i=1}^{n}\sigma_i^{el}(t)$$

(pseudostrain)

$$\left.\frac{\mathrm{d}C}{\mathrm{d}S}\right|_{@\,t} = \exp(a\,\mathrm{S}(t)^{b})\,a\,b\,\mathrm{S}(t)^{b-1}$$

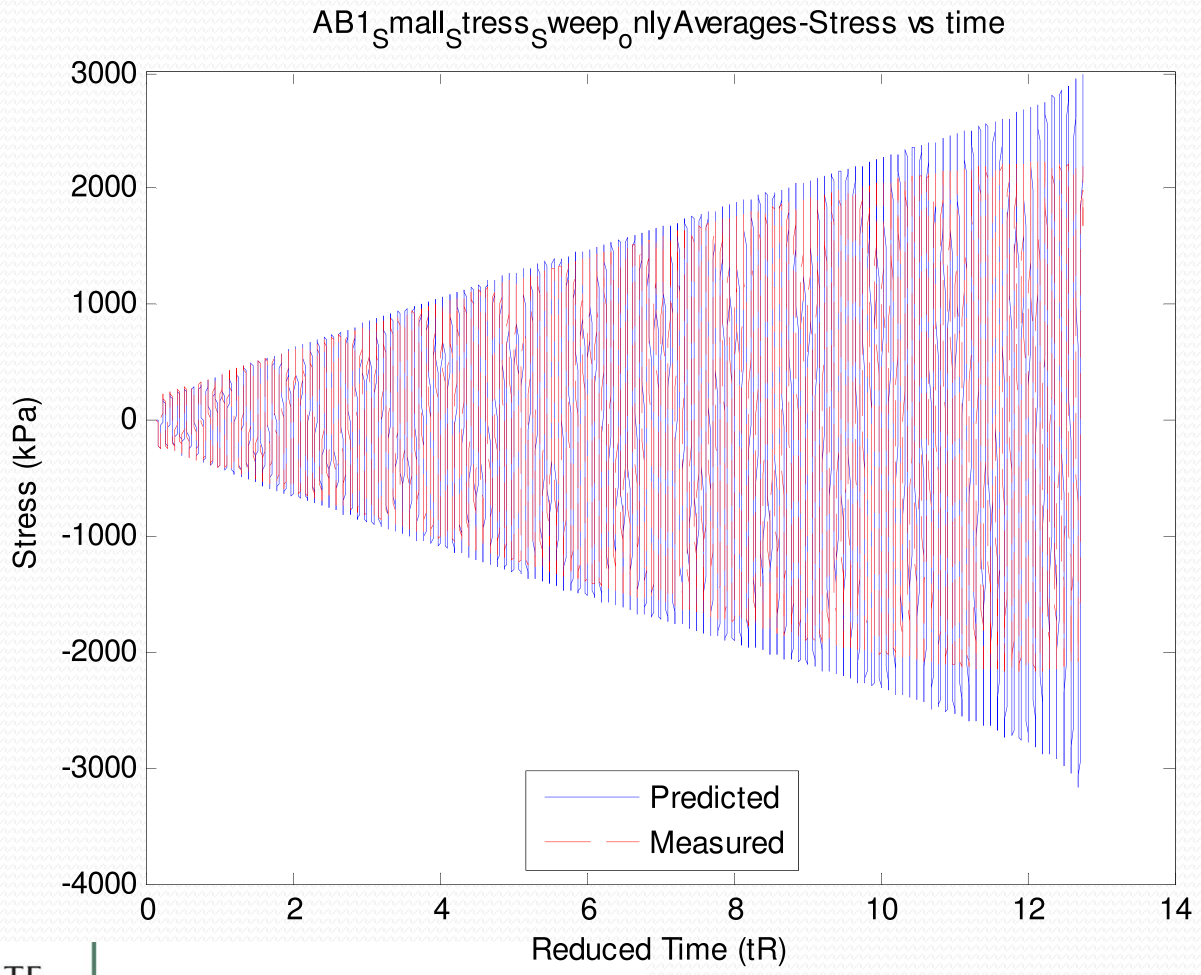
$$S(t+\Delta t) = S(t) + \Delta t\left[-0.5\,I\,\varepsilon^{\mathrm{R}}(t)^{2}\left.\frac{\mathrm{d}C}{\mathrm{d}S}\right|_{@\,t}\right]^{\alpha}$$

$$\mathrm{C}(t+\Delta t) = \exp(a\,\mathrm{S}(t+\Delta t)^{b})$$

$$\sigma(t+\Delta t) = I\,\mathrm{C}(t+\Delta t)\ \varepsilon^{\mathrm{R}}(t+\Delta t)$$

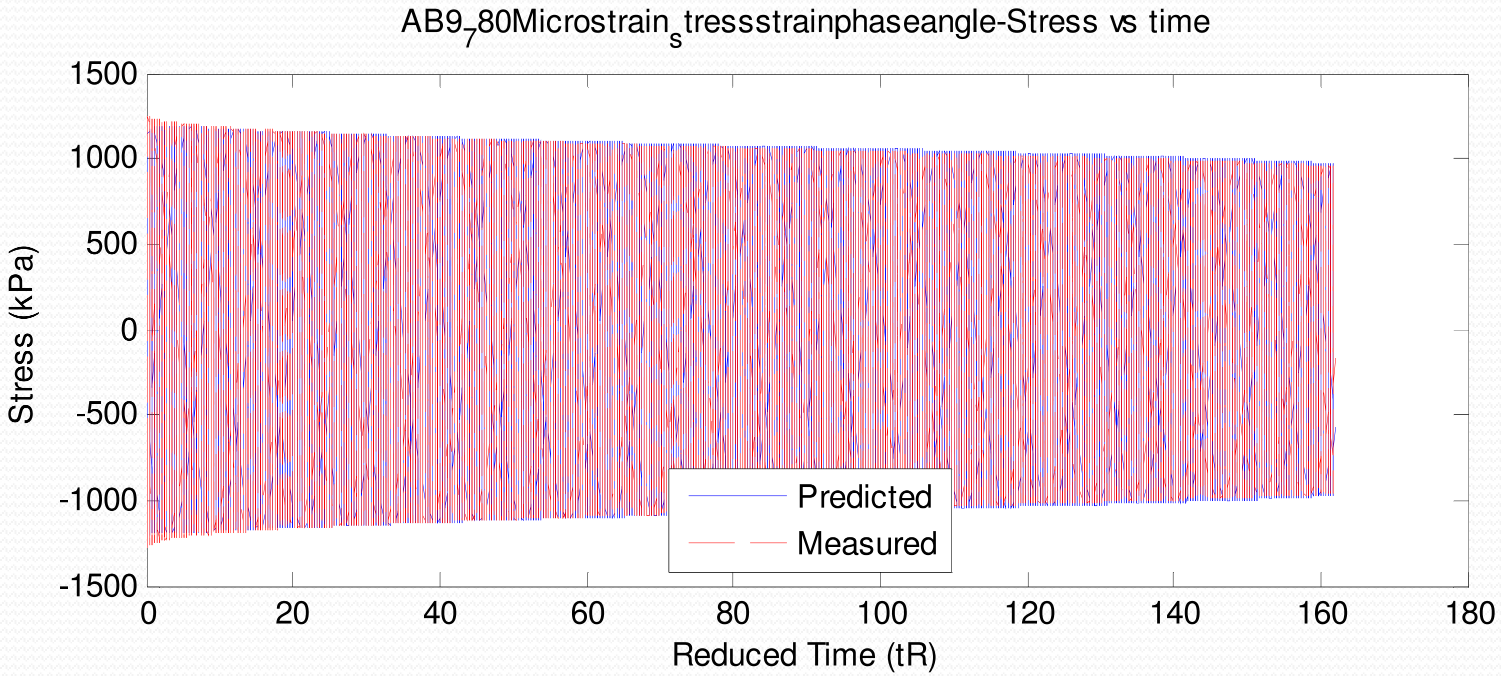
Validation methodology

STEP (3) Validation (A) Stress sweep testing at 10Hz, 19C



Validation methodology

STEP (3) Validation (B) crosshead strain controlled fatigue testing at 10Hz, 19C

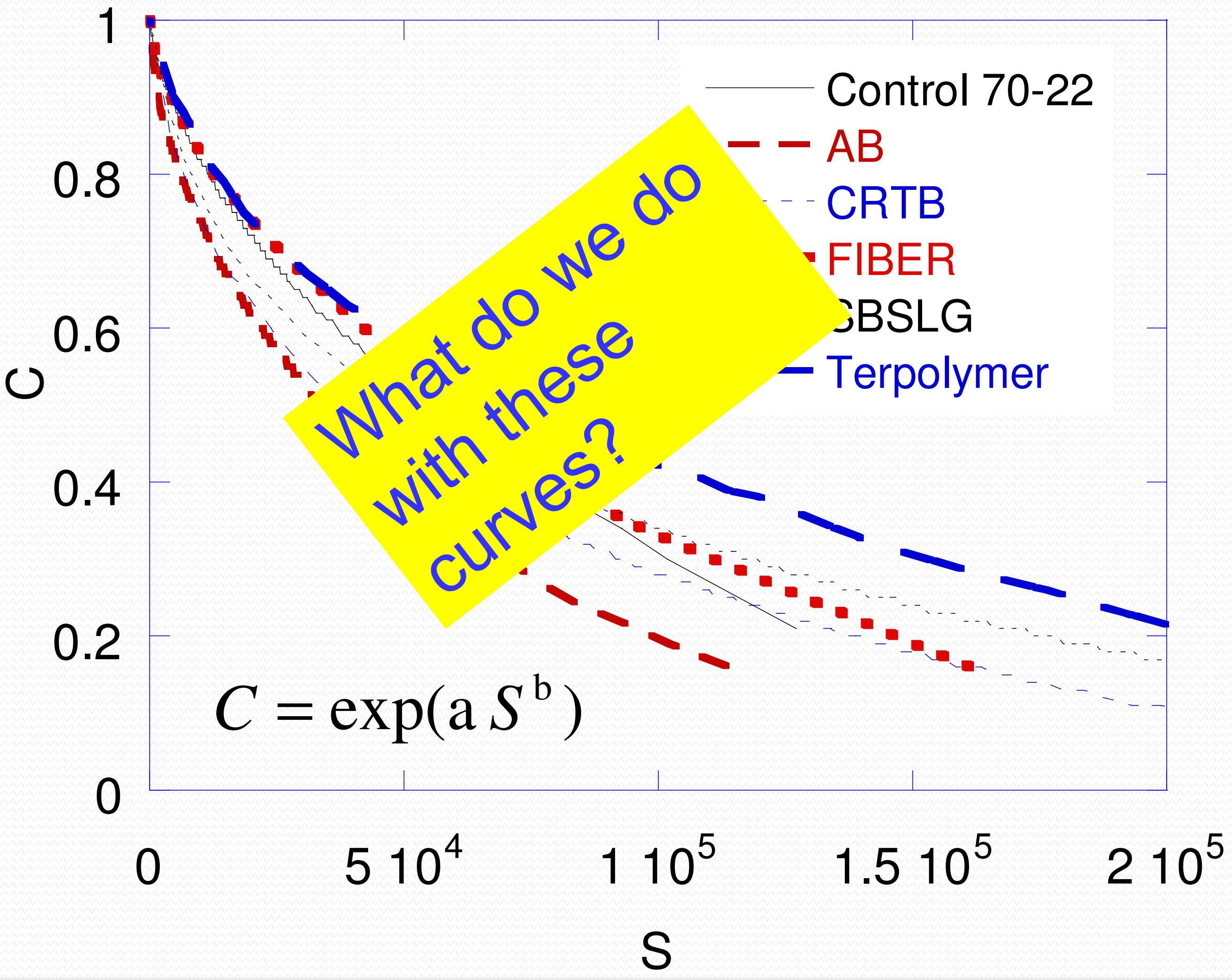


Is peak to peak C vs S same as the C vs S computed using the hereditary integral ?

Answer: YES!

C vs S curves of all mixtures

$$C = \frac{|E^*|_N}{|E^*|_{LVE}}$$



What do we do with these curves?

$$C = \exp(a\,S^{b})$$

$$S_{N+\Delta N} = S_N + (\Delta N/f)^{\frac{1}{1+\alpha}}\left[-0.5\,I\,\varepsilon_N^{R\,2}(C_{N+\Delta N} - C_N)\right]^{\frac{\alpha}{1+\alpha}}$$

USE #1:

Finite Element Implementation

Research led by R. Kim at NCSU with ALF materials

U.S. Department of Transportation

Federal Highway Administration

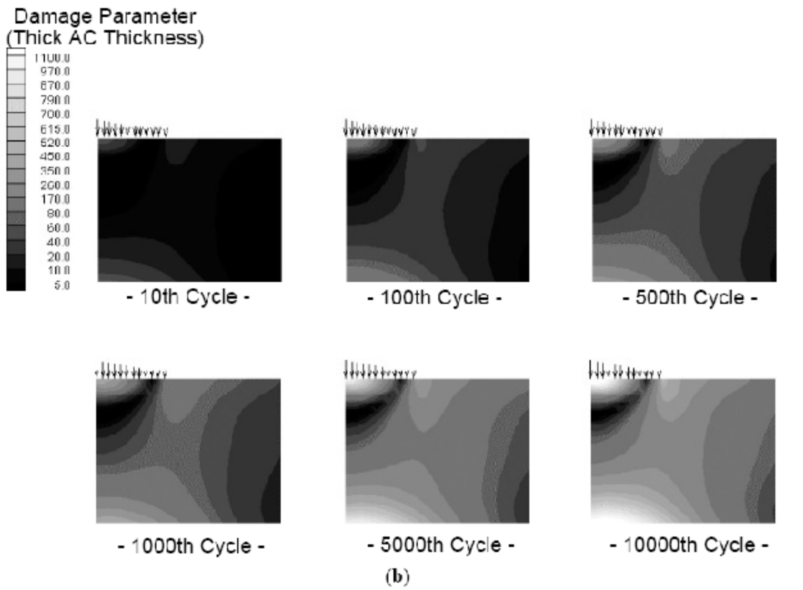
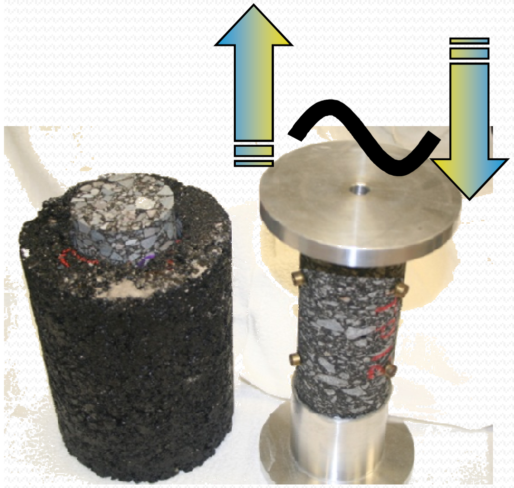
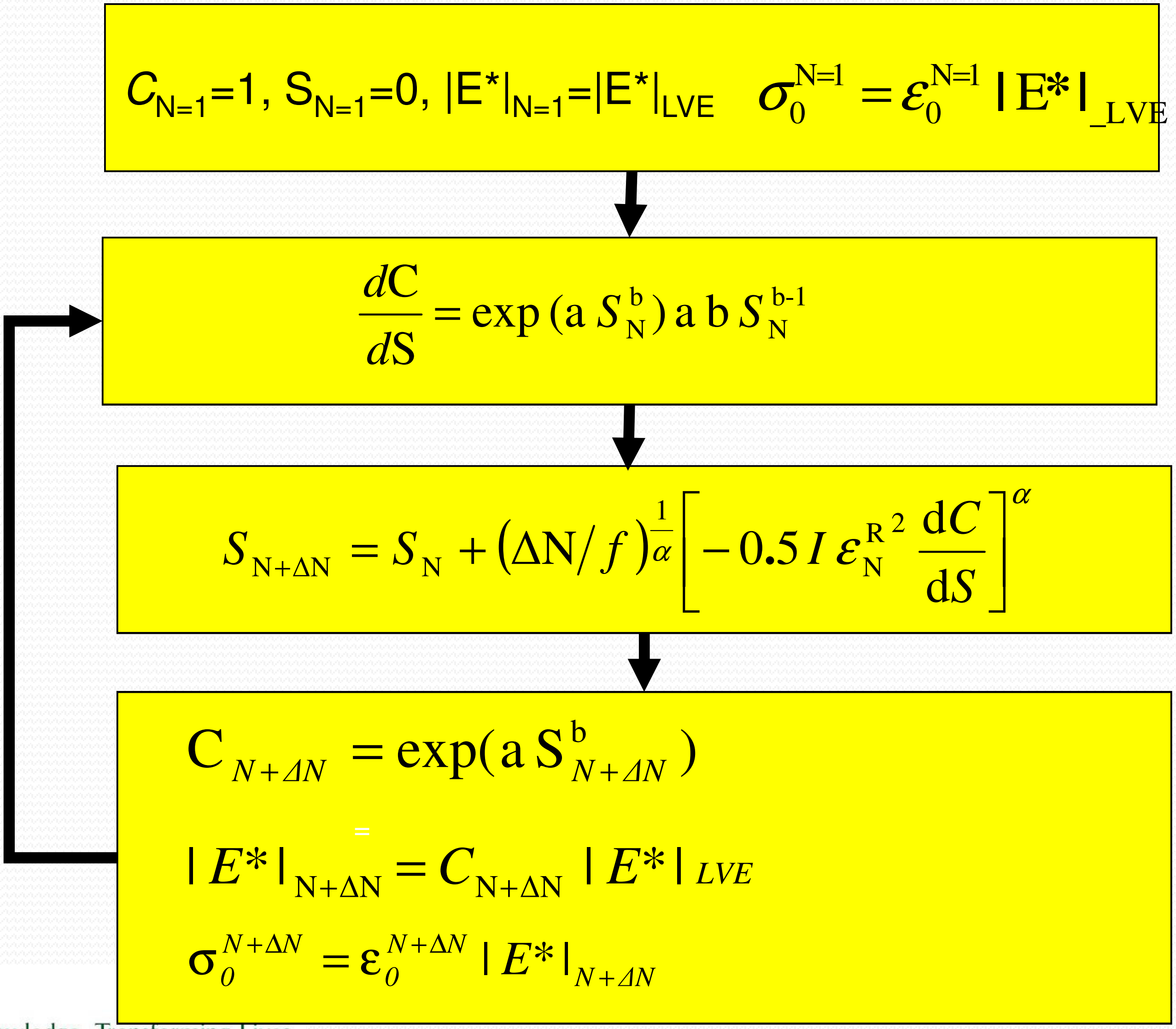


FIGURE 3 VECD-FEP++ analysis of a pavement with thick asphalt layer: (*a*) cyclic hysteresis behavior and (*b*) damage contours at different cycles.

USE #2 →Simulation of uniaxial cyclic strain controlled tests (Simplified & More Practical)





$C_{N=1}=1$, $S_{N=1}=0$, $|E^*|_{N=1}=|E^*|_{LVE}$ $\quad \sigma_0^{N=1}=\varepsilon_0^{N=1}\,|E^*|_{\_LVE}$

$$\frac{dC}{dS}=\exp\left(a\,S_N^{b}\right)a\,b\,S_N^{b-1}$$

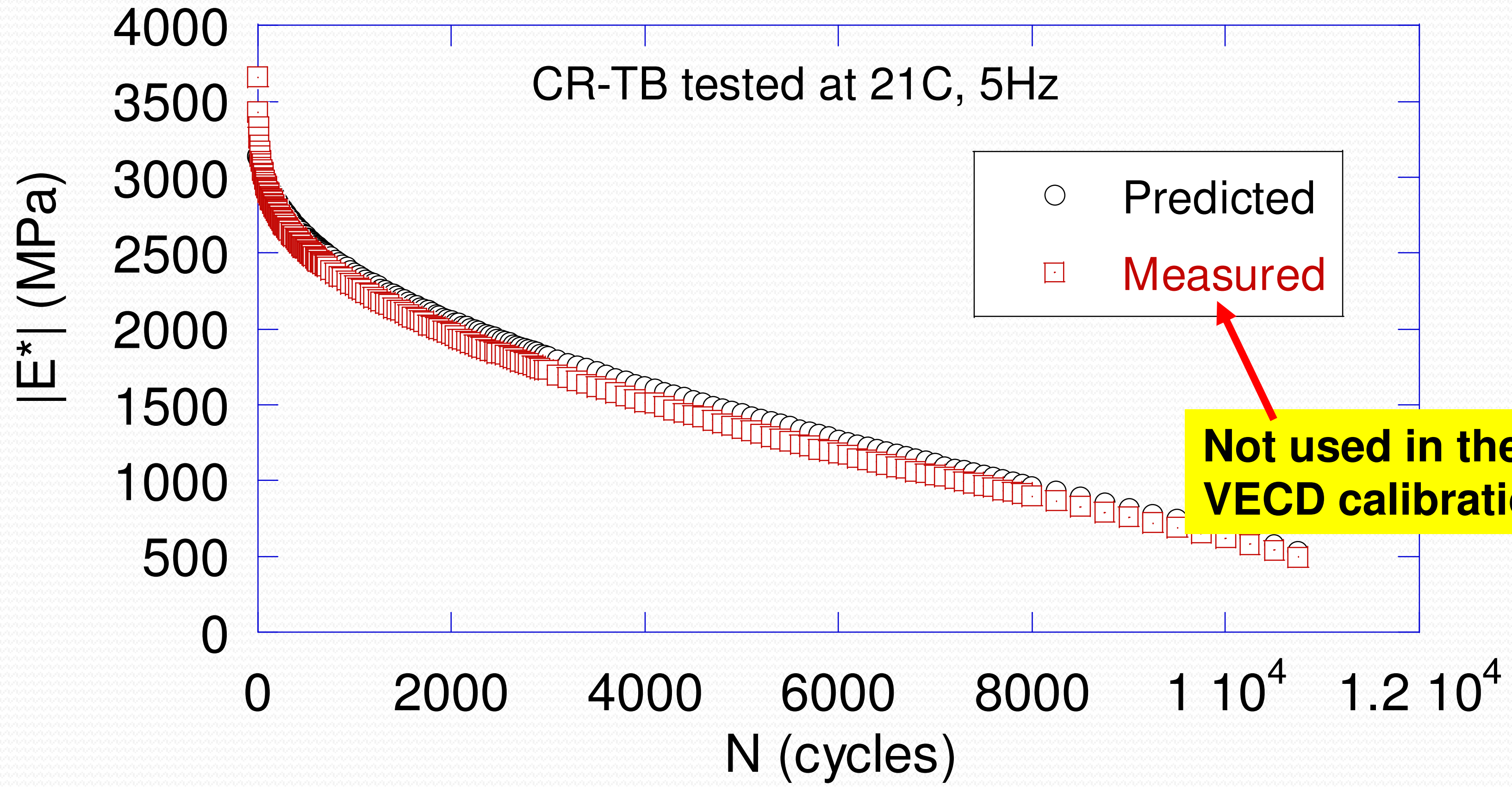
$$S_{N+\Delta N}=S_N+\left(\Delta N/f\right)^{\frac{1}{\alpha}}\left[-0.5\,I\,\varepsilon_N^{R\,2}\,\frac{dC}{dS}\right]^{\alpha}$$

$$C_{N+\Delta N}=\exp\left(a\,S^{b}_{N+\Delta N}\right)$$

$$|E^*|_{N+\Delta N}=C_{N+\Delta N}\,|E^*|_{LVE}$$

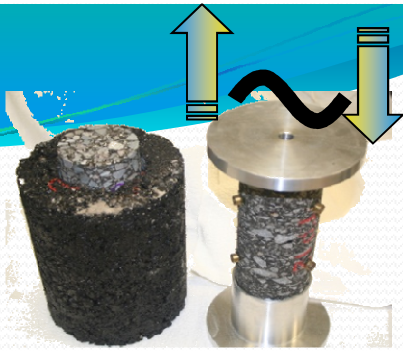
$$\sigma_0^{N+\Delta N}=\varepsilon_0^{N+\Delta N}\,|E^*|_{N+\Delta N}$$

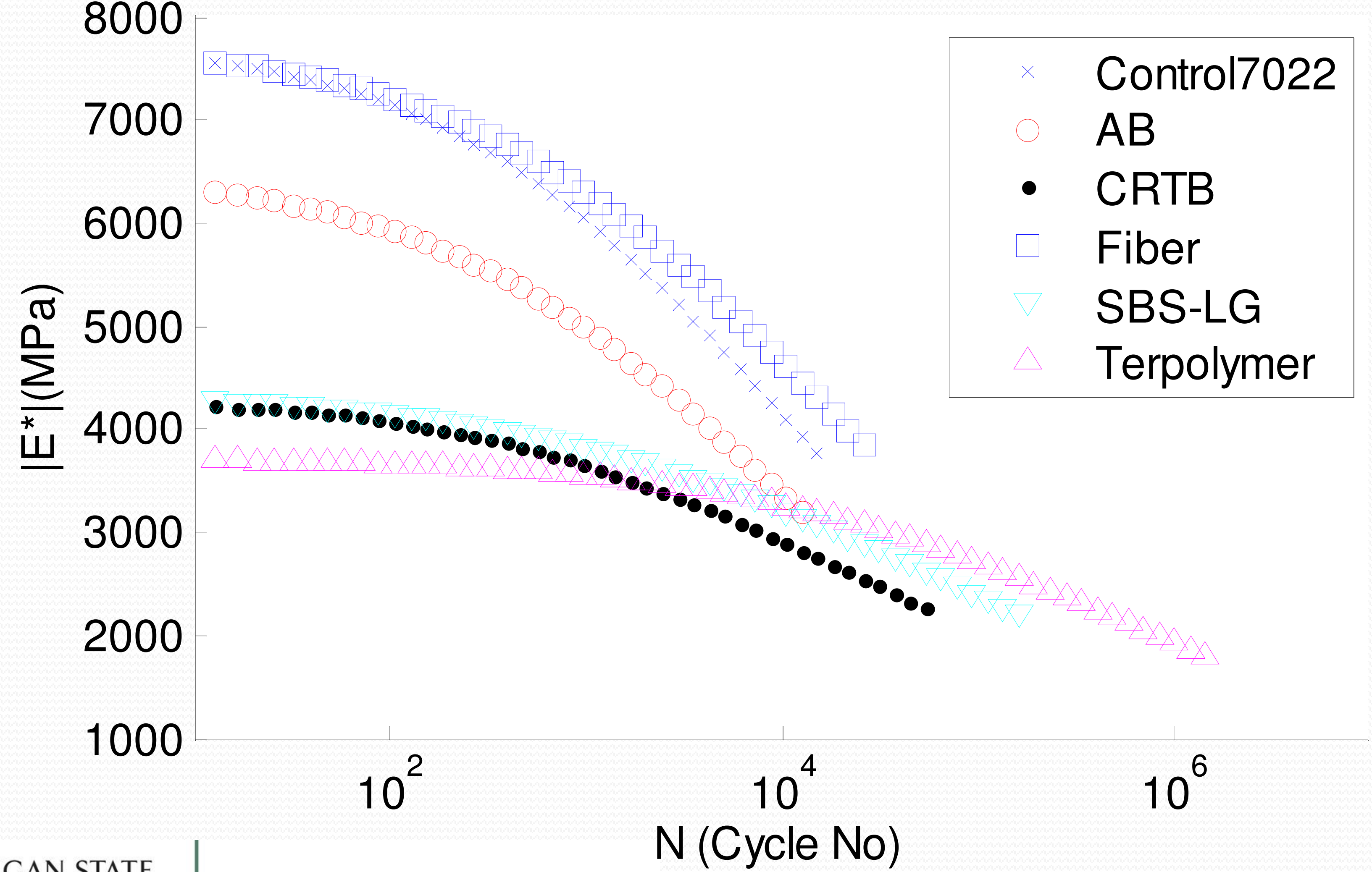
Validation of the applicability of VECD to push-pull fatigue tests



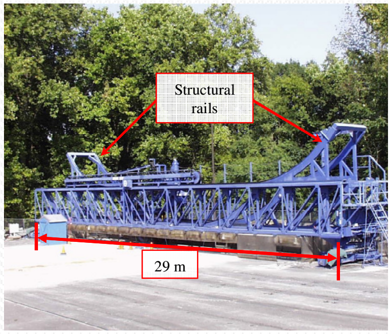
Not used in the VECD calibration

Push-pull fatigue simulation results (ALF Mixtures)

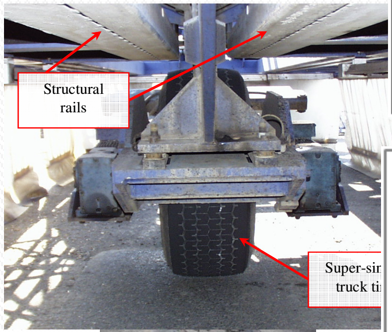




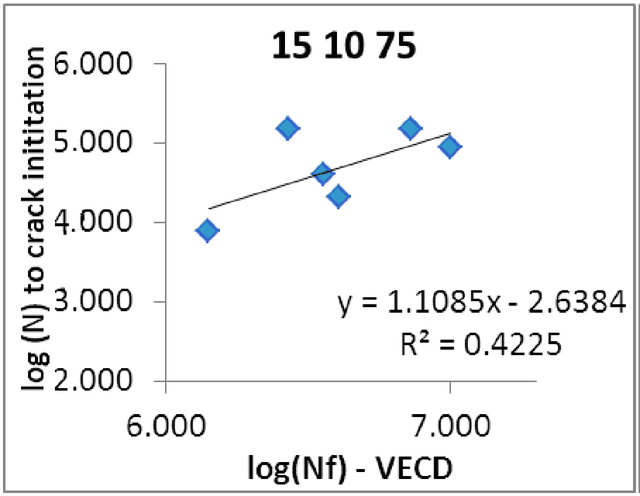
Comparison with field accelerated pavement testing data

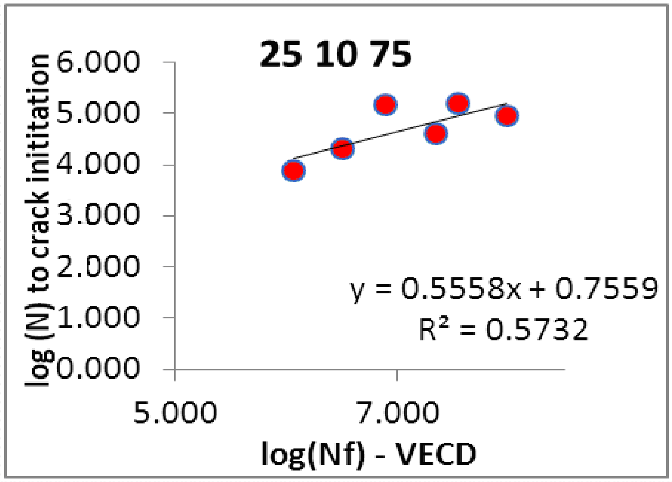


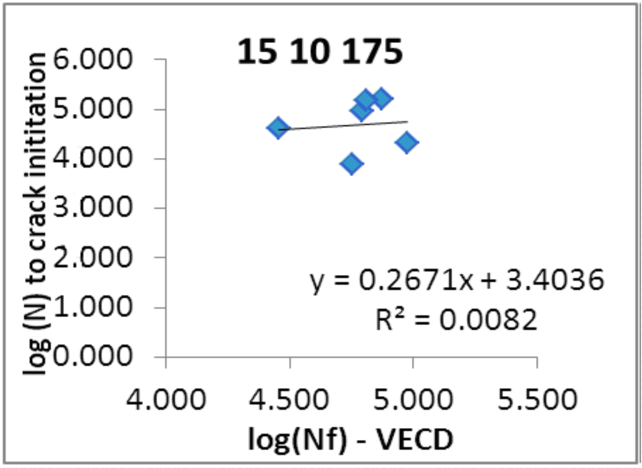
a

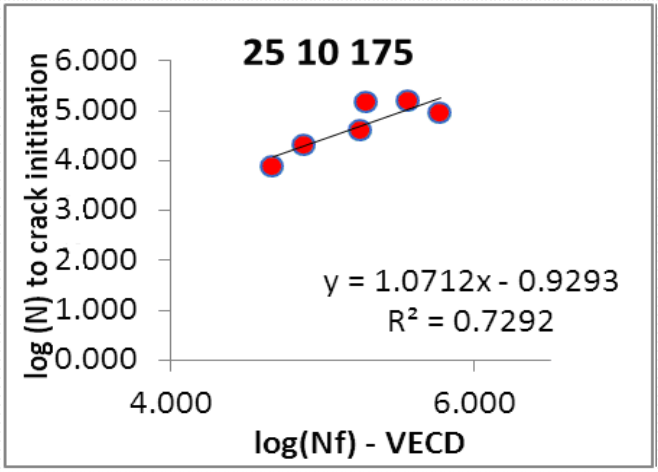


b







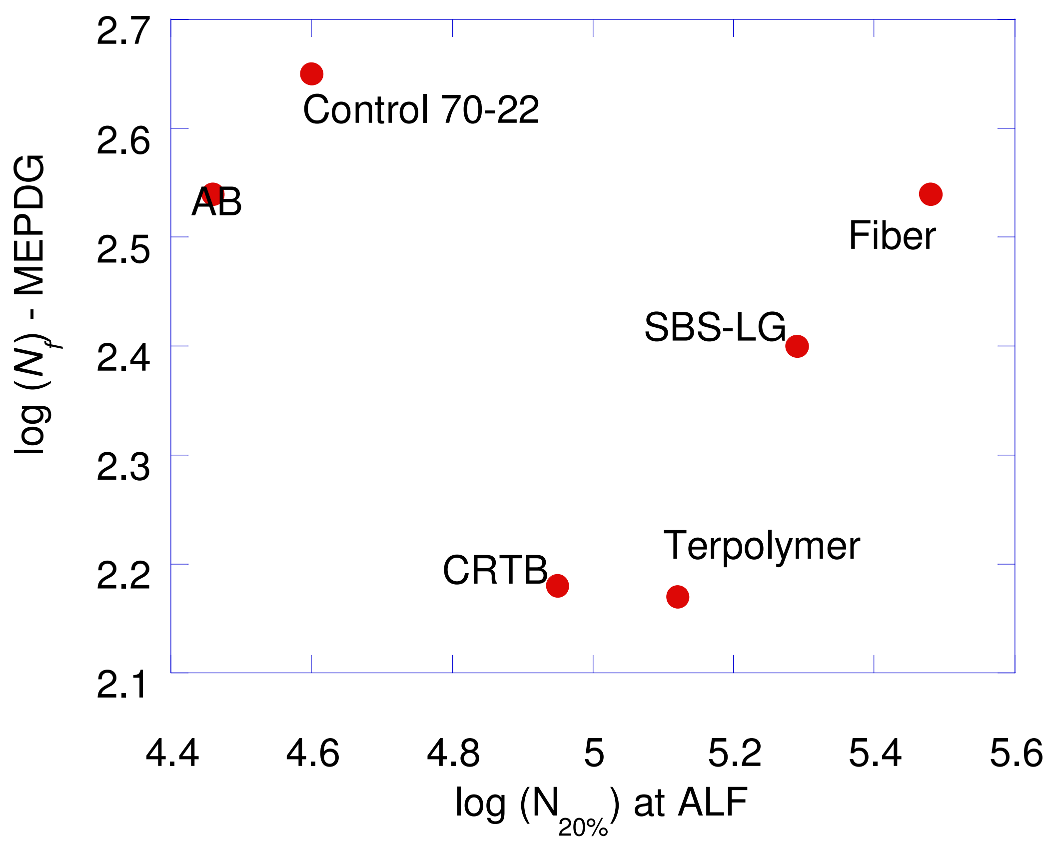


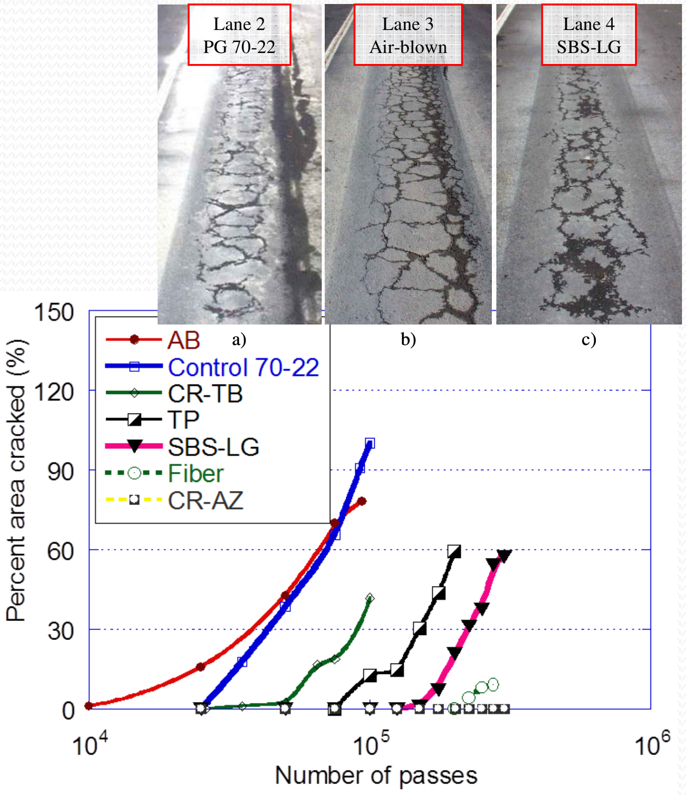
Use #3: Fatigue life from the VECD and proposed MEPDG implementation

N_f in MEPDG vs. ALF cracking

$$N_f = (16.6)\, C_H\, C \left(\frac{1}{\varepsilon_t}\right)^{0.8939} \left(\frac{1}{E}\right)^{0.0779}$$

- |E*| at 19°C 2.5Hz
- Used the bottom measured strain
- Local calibration constants





Number of cycles to failure (N_f): General form of the equation

$$\frac{dS}{dN} f = \left[-\frac{\varepsilon^{R\,2}}{2}\frac{dC}{dS}\right]^{\alpha}$$

$$\left[-\frac{\varepsilon^{R\,2}}{2}\frac{dC}{dS}\right]^{-\alpha} f\,dS = dN$$

$$\int_0^{S_f}\left[-\frac{\varepsilon^{R\,2}}{2}\frac{dC}{dS}\right]^{-\alpha} f\,dS = \int_0^{N_f} dN$$

$$N_f = \int_0^{S_f}\left[-\frac{\varepsilon^{R\,2}}{2}\frac{dC}{dS}\right]^{-\alpha} f\,dS$$

$$\varepsilon^R = |E^*|_{LVE}\ \varepsilon_0$$

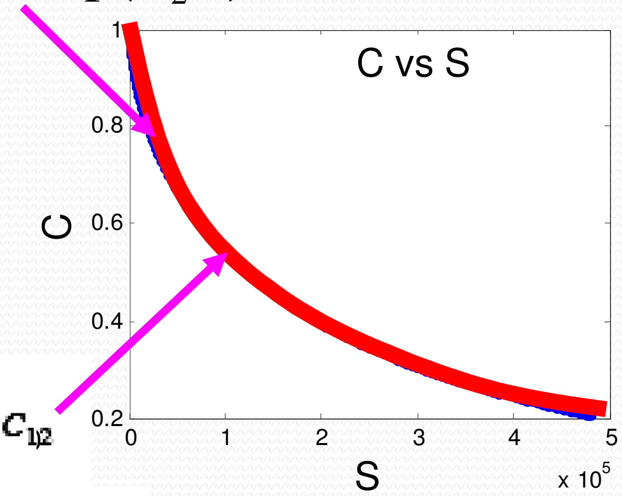
$$N_f = \sum_{S=1}^{S_f}\left[-\frac{\varepsilon_0^2 \left|E^*\right|_{LVE}^2}{2}\left.\frac{dC}{dS}\right|_{at\,S}\right]^{-\alpha} f\,\Delta S_S$$

Reference: Kutay et al. TRB 2009

Closed-form solutions of the N_f equation for special cases

- *Christensen & Bonaquist →

$$C = \exp(C_2 S)$$



$$N = \frac{2^{\alpha} f\left(C^{-\alpha} - 1\right)}{\alpha(-C_2)^{1+\alpha} \varepsilon_0^{2\alpha} \left|E^*\right|_{LVE}^{2\alpha}}$$

- **Lee et al. →

$$C_1(S_{1n}) = C_{10} - C_{11}(S_{1n})^{C_{12}}$$

$$N_{f,w/oRP} = \frac{f(S_{1f})^{p_1}}{p_1(0.125IC_{11}C_{12})^{\alpha_1}} \left|E^*\right|^{-2\alpha_1} \varepsilon_0^{-2\alpha_1}$$

* **Christensen, D. W. and Bonaquist, R. F. (2005).** "Practical application of continuum damage theory to fatigue phenomena in asphalt concrete mixtures." *J. Assn. of Asphalt Paving Technologists*, Vol.74, pp. 963-1002.

** **Lee, H. J., Daniel J. S., and Kim, Y. R. (2000)**"Continuum damage mechanics–based fatigue model of asphalt concrete." *J. Mater. Civ. Eng.*, 12(2), 105–112.

General form of the VECD-N_f equation

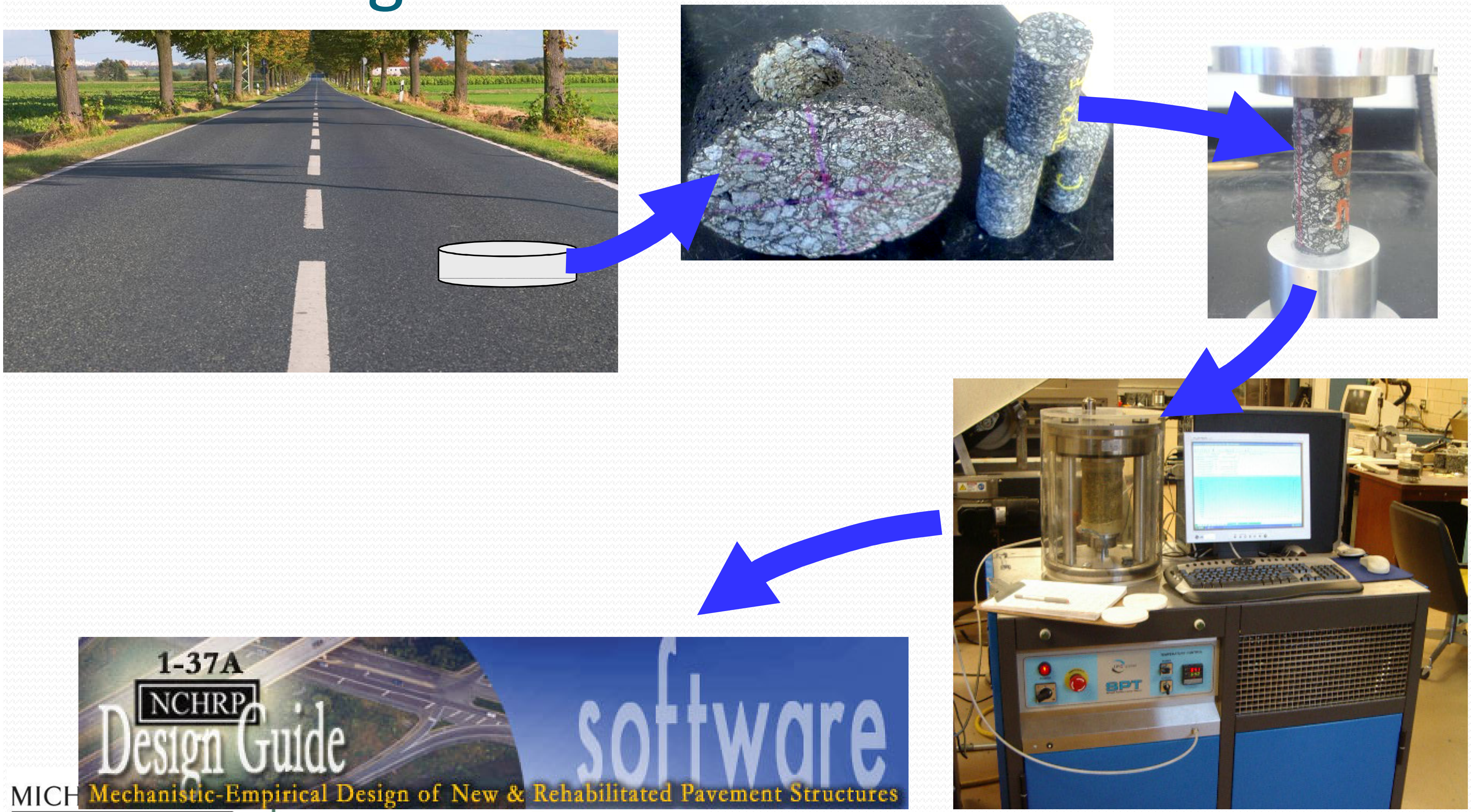
$$N_f = \sum_{S=1}^{S_f} \left[-\frac{\varepsilon_0^2 \left|E^*\right|_{LVE}^2}{2} \left.\frac{dC}{dS}\right|_{at\,S} \right]^{-\alpha} f\,\Delta S_S$$

- Procedure:
  - Select the C(S) function that best fits to given data
  - Select failure criterion, e.g., C=0.5, strain level (ε_0) and $|E^*|_{LVE}$
  - Calculate S_f corresponding to C=0.5
  - Calculate N_f using equation above.

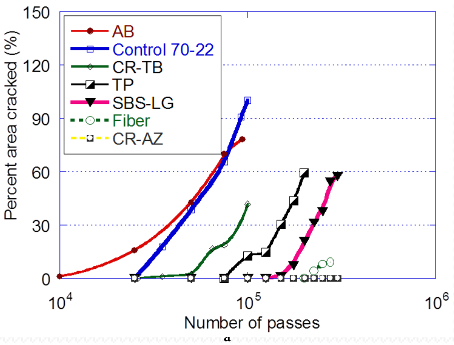
Proposed MEPDG implementation

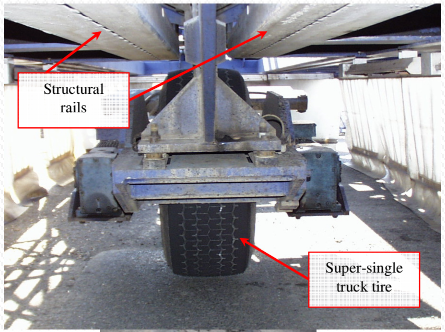
- Level 1 input (using AMPT)
  - |E*| master curve
  - Push-pull test at a specified temperature and frequency (e.g., 15^{o}C, 10Hz)

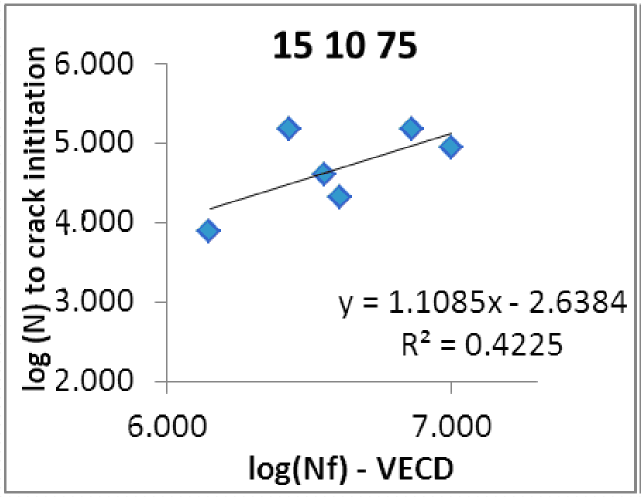
Possible use of VECD in MEPDG for remaining service life

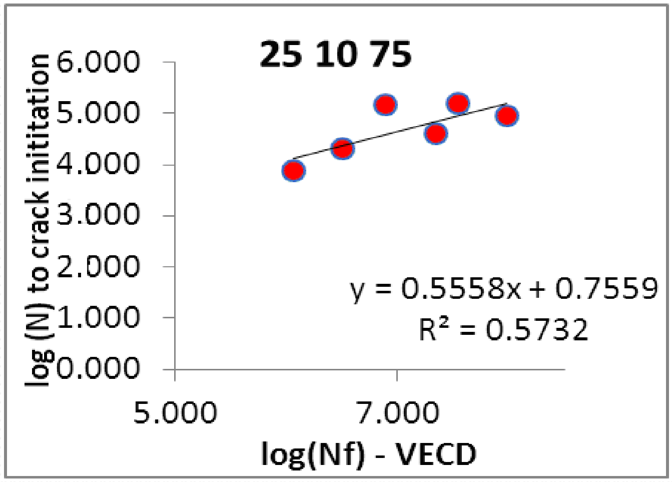


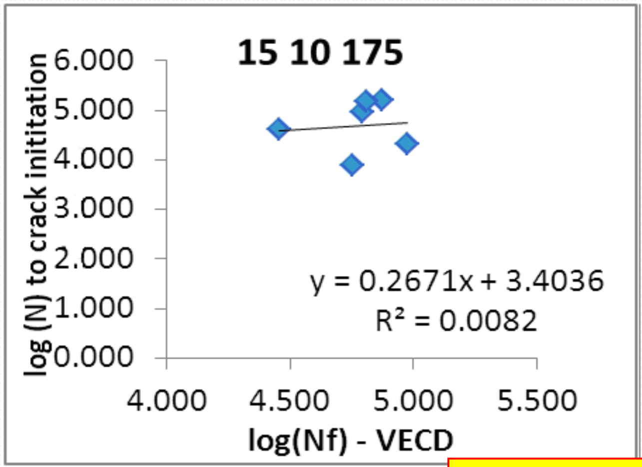
Comparison with field accelerated pavement testing data

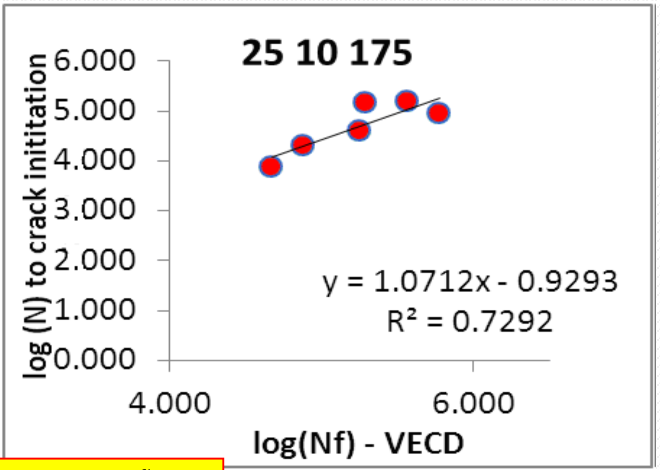












$$N_f = \sum_{S=1}^{S_f} \left[-\frac{\varepsilon_0^2 \left|E^*\right|_{LVE}^2}{2} \left.\frac{dC}{dS}\right|_{at\,S} \right]^{-\alpha} f\,\Delta S_s$$

The End

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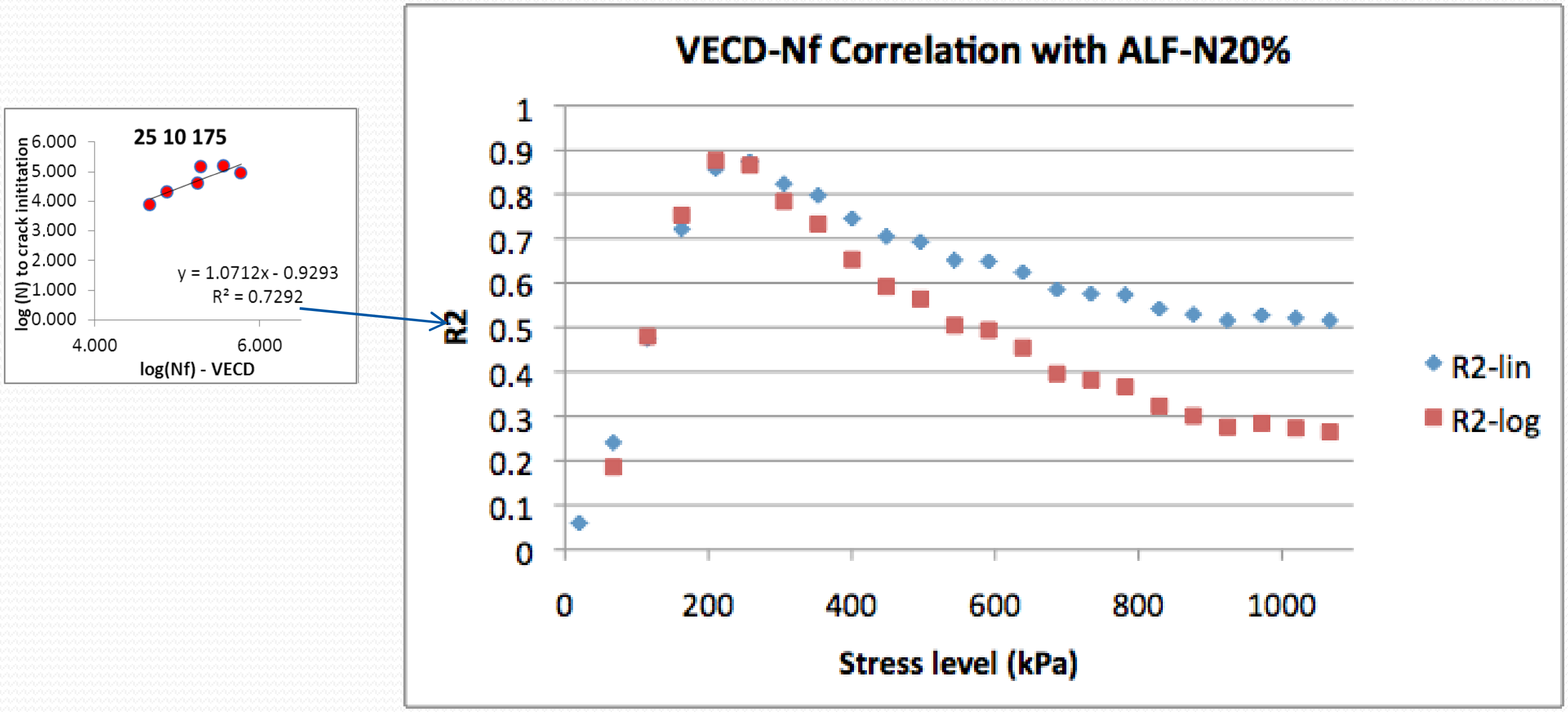
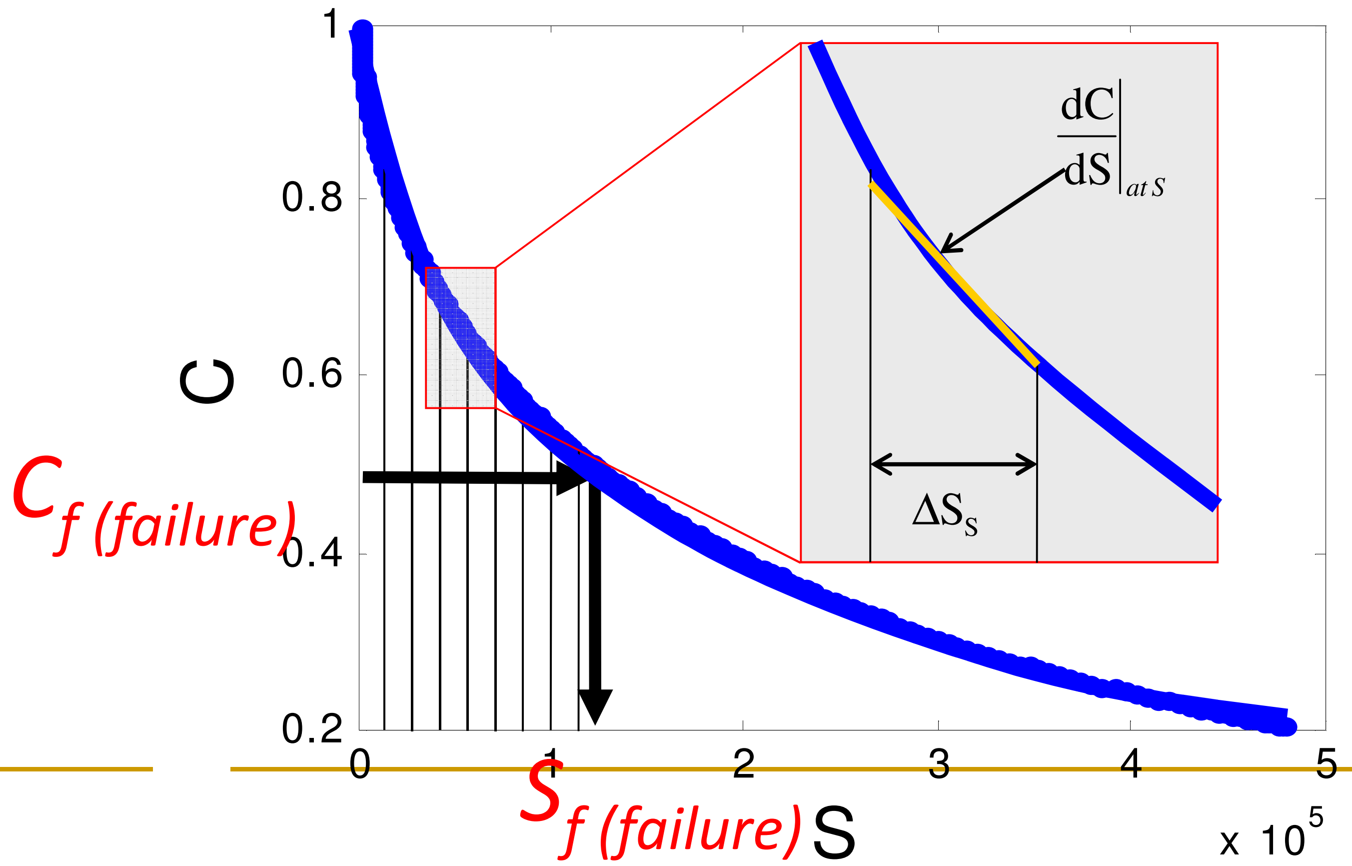
Correlation of VECD-Nf with field- different load levels:



Prediction of fatigue life

$$N_f = \sum_{S=1}^{S_f} \left[-\frac{\varepsilon_0^2 \left|E^*\right|_{LVE}^2}{2} \left.\frac{dC}{dS}\right|_{at\,S} \right]^{-\alpha} f\,\Delta S_S$$



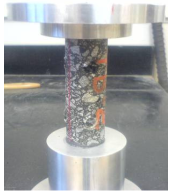
Specimen size limitation

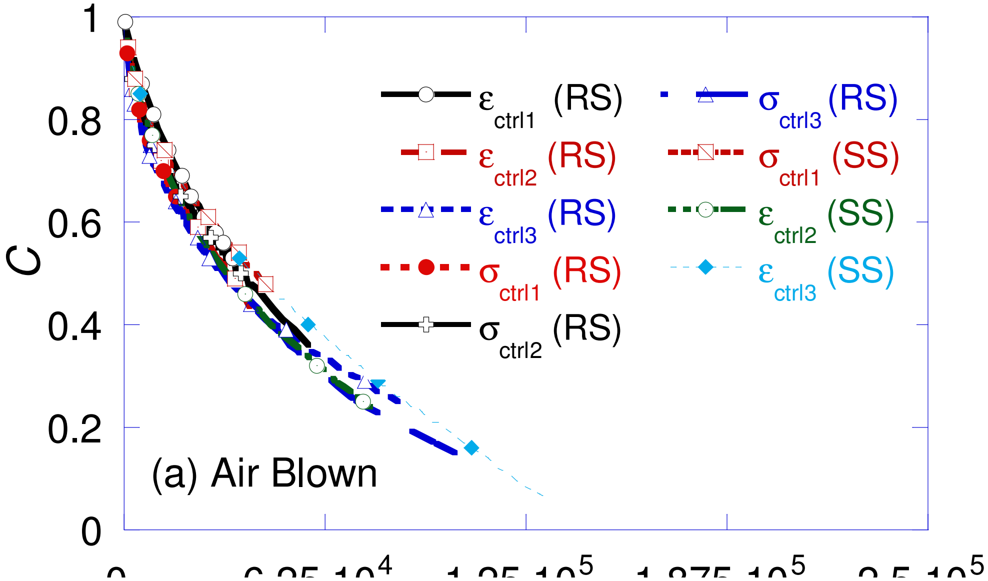
- Traditional specimen sizes:
  - 100 or 75 mm diameter, 150 mm tall

- Thin pavements (thickness< 150mm) are not suitable for field core testing
- Solution (?)
  - Small diameter & small height samples
  - Horizontal coring from the field slabs

Can small size samples work?

- Regular Size (RS)→ D = 71.4 mm, H =150 m
- Small Size (SS) →D = 38.1 mm , H= 100 mm



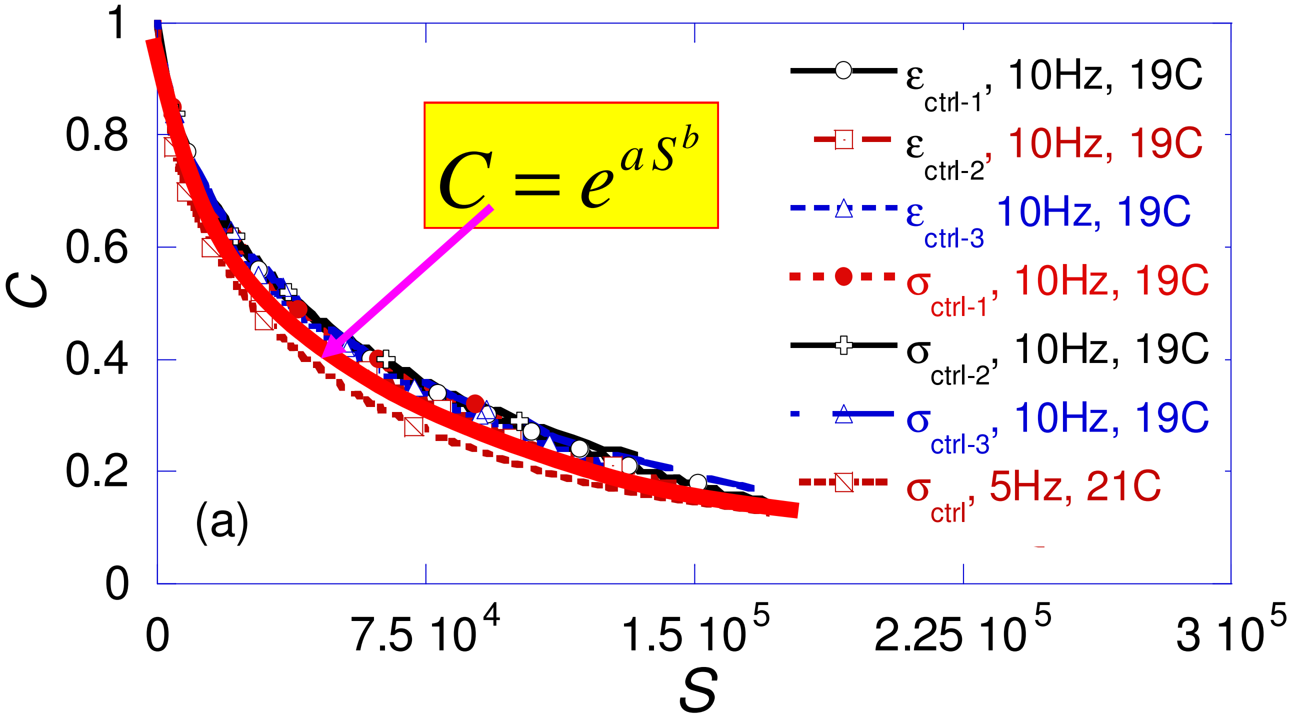


Answer: Yes!

Reference: Kutay, Gibson & Youtcheff @ TRB 2009

Next step after obtaining C & S

- Develop the relationship by fitting a simple equation



$$C = e^{aS^b}$$