Models for Plastic Deformation Based on Non-Linear Response for FEM Implementation

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Outline

Motivation

Viscoelastic-Viscoplastic-Viscodamage Models

Identification of model parameters and model validation

Implementation procedure

Application of the model for rutting performance simulation

Conclusions and future works
Motivation

Prediction of pavement performance

Material Characteristics
- Aggregate shape
- Anisotropy
- Adhesive and Cohesive bond strengths
- Healing
- Binder rheology

Pavement Structure
- Thickness
- Sub grade type

Environmental Factors
- Temperature
- Humidity differential
- Rainfall

Loading Conditions
- Type of loading
- Tire pavement interaction

FEM Modeling

Calibration (minimize shift factor dependency)

Prediction of material distress
Mechanical Response of Asphalt Mixes

Viscoelastic-Viscoplastic-Viscodamage

- Temperature Increase and/or decrease in strain rate
- Percent of Strain
- Strain
- Time
Mechanical Response of Asphalt Mixes

Viscoelastic-Viscoplastic-Viscodamage

Rate- and Time-dependent softening

Displacement control tests
Repeated creep-recovery test
Outline

Viscoelastic-Viscoplastic-Viscodamage Models
Viscoelastic Properties

Nonlinear Viscoelastic Model (Schapery, 1969)

Nonlinear contribution in the transient response

\[ \epsilon^{ve} = g_0 D_0 \sigma + g_1 \int_0^t D \left( \psi^t - \psi^{\tau} \right) \frac{d}{d\tau} g_2 \sigma \, d\tau \]

Nonlinear contribution in the elastic response, due to the level of stress

\[ \psi = \int_0^t \frac{dt}{a_T} \]

Nonlinear contribution in the viscoelastic response due to the level of stress

\[ D = \sum_{n=1}^N D_n \left[ 1 - \exp \left( -\frac{t}{\lambda_n} \right) \right] \]

Temperature shift factor

Prony series coefficients

Retardation time
Viscoplastic Properties

Viscoplastic Model (Perzyna, 1971)

\[ \Delta \varepsilon_{ij} = \Delta \varepsilon_{ij}^{ve} + \Delta \varepsilon_{ij}^{vp} \quad \text{Perzyna Model} \]

\[ f = F(\sigma_{ij}) - \kappa(\varepsilon_{e}^{vp}) = \tau - \alpha I_1 - \kappa(\varepsilon_{e}^{vp}) \quad \text{Yield surface (Extended Drucker-Prager)} \]

\[ \kappa = \kappa_0 + \kappa_1 \left[ 1 - \exp\left(-\kappa_2 \varepsilon_{e}^{vp}\right) \right] \quad \text{Hardening function} \]

Flow Rule: \[ \dot{\varepsilon}_{ij}^{vp} = \Gamma^{vp}(T) \langle \Phi(f) \rangle^N \frac{\partial g}{\partial \sigma_{ij}} \]

Plastic potential function: \[ g = \tau - \beta I_1 \]

Overstress function \[ \langle \phi(f) \rangle = \begin{cases} 0 & f \leq 0 \\ f & f > 0 \end{cases} \]

\[ \Phi(f) \quad \text{Viscoplastic} \quad f > 0 \]

\[ \text{Viscoelastic} \quad f \leq 0 \]

\[ \text{Yield surface} \]
Extended Drucker-Prager Yield Surface

Accounts for:

- Dilation and confinement pressure
- The effect of shear stress
- Work hardening of the material

\[ f = \tau - \alpha I_1 - \kappa \]
\[ g = \tau - \beta I_1 - \chi \]
Viscoplastic Properties

Effect of parameter “d” on the yield surface

\[ \tau = \sqrt{J_2} \left[ 1 + \frac{1}{d} - \left( 1 + \frac{1}{d} \right) \frac{J_3}{J_2^{3/2}} \right] \]

Influence of stress path on the yielding point
Strength Degradation due to Damage

\[ \sigma = (1 - \phi) \bar{\sigma} \]

\[ \phi = \frac{A - \bar{A}}{A} \]

0 ≤ \( \phi \) ≤ 1

<table>
<thead>
<tr>
<th>( \sigma )</th>
<th>( \phi ) = 0</th>
<th>No damage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal (measured) stress</td>
<td>( \phi ) = 1</td>
<td>Failure</td>
</tr>
<tr>
<td>Effective stress</td>
<td>0 ≤ ( \phi ) ≤ 1</td>
<td>Damage</td>
</tr>
<tr>
<td>Damage density</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Viscodamage Model

Viscodamage model (Darabi, Abu Al-Rub, Masad, Little; 2010)

\[ \dot{\phi} = \Gamma^{vd} \left( \frac{Y}{Y_0} \right)^q \left( 1 - \phi \right)^2 \exp(k \varepsilon^{Tot}) \exp \left[ -\delta \left( 1 - \frac{T}{T_0} \right) \right] \]

Damage force

\[ Y = \frac{\sqrt{J_2}}{2} \left[ 1 + \frac{1}{d} - \left( 1 + \frac{1}{d} \right) \frac{J_3}{J_2^{3/2}} \right] - \alpha I_1 \]

Damage is sensitive to hydrostatic pressure

Damage response is different in compression or extension

Damage is sensitive to Loading Mode

\( I_1 \) : First stress invariant

\( J_2 \) and \( J_3 \) : The second and the third deviatoric stress invariants
## Determination of Model Parameters

<table>
<thead>
<tr>
<th>Procedure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Creep-Recovery test @ reference temperature</td>
<td>Separate viscoelastic (recoverable) and viscoplastic (irrecoverable) strains.</td>
</tr>
<tr>
<td>Recovery part of the Creep-Recovery test @ reference temperature</td>
<td>Determination of viscoelastic parameters @ reference temperature</td>
</tr>
<tr>
<td>Creep part of the Creep-Recovery test @ reference temperature</td>
<td>Determination of viscoplastic parameters @ reference temperature</td>
</tr>
<tr>
<td>Two creep tests that show tertiary behavior @ reference temperature</td>
<td>Determination of viscodamage parameters @ reference temperature</td>
</tr>
<tr>
<td>Creep test in tension @ reference temperature</td>
<td>Determination of “$d$” parameters</td>
</tr>
<tr>
<td>Creep-recovery and creep tests @ other temperatures</td>
<td>Determination temperature-dependent model parameters</td>
</tr>
</tbody>
</table>
Determination of Model Parameters

Creep-Recovery test @ reference temperature

Separate viscoelastic (recoverable) and viscoplastic (irrecoverable) strains.

\[ \varepsilon(t_a) = g_0 D_0 \sigma + g_1 g_2 D(t_a) \sigma + \varepsilon^{vp}(t_a) \]

\[ \varepsilon(t) = g_1 g_2 [D(t) - D(t-t_a)] \sigma + \varepsilon^{vp}(t) \]

During the recovery:

\[ \varepsilon^{vp}(t_a) = \varepsilon^{vp}(t) \]

\[ \varepsilon(t_a) - \varepsilon(t) = g_0 D_0 \sigma + g_1 g_2 [D(t_a) - D(t) + D(t - t_a)] \sigma \]

Pure viscoelastic response.
Can be used for identifying VE parameters
Determination of Model Parameters

Creep part of the Creep-Recovery test @ reference temperature

Determination of viscoplastic parameters @ reference temperature

- Viscoelastic response. Model predictions using the Identified VE model parameters.
- Total strain (experimental measurements)
- Time
- Strain

Viscoplastic response. Obtained by subtracting the VE response from the experimental measurements

Pure viscoplastic response. Can be used for identifying VP parameters
Determination of Model Parameters

A creep test that shows tertiary behavior @ reference temperature and stress level

Determination of viscodamage parameters @ reference temperature

Total strain (experimental measurements) @ reference temperature and stress level

Deviation from the experimental data at tertiary stage due to damage

Model response using identified VE-VP model parameters

@ reference temperature $T=T_0$

$\exp \left[ -\delta \left( 1 - \frac{T}{T_0} \right) \right] = 1$

@ reference stress $Y=Y_0$

$\phi = \Gamma^{vd} (1 - \phi)^2 \exp \left( k \varepsilon^{Tot} \right)$

Identify these damage parameters
Using the creep test at reference temperature and stress level
Determination of Model Parameters

Another creep tests that show tertiary behavior @ reference temperature and other stress level

Determination of viscodamage parameters @ reference temperature

Model response using identified VE-VP-VD model parameters

Deviation from the time of failure due to stress dependency parameter “q”

Total strain (experimental measurements) @ reference temperature and stress level

Identify the stress dependency parameter “q”

\[ \phi = \Gamma_{vd} \left( \frac{Y}{Y_0} \right)^{q} (1 - \phi)^2 \exp \left( k \varepsilon^{Tot} \right) \]

@ reference temperature \( T = T_0 \)

\[ \exp \left[ -\delta \left( 1 - \frac{T}{T_0} \right) \right] = 1 \]

Known
Stress Levels within the Pavements

Gibson et al., 2010
# Model Validation Tests

<table>
<thead>
<tr>
<th>Test</th>
<th>Temperature (°C)</th>
<th>Stress Level (kPa)</th>
<th>Loading time (Sec)</th>
<th>Strain Rate (1/Sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Compression</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Creep-Recovery</td>
<td>10</td>
<td>2000, 2500</td>
<td>300, 350, 400</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>1000, 1500</td>
<td>30, 40, 130, 210</td>
<td></td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>500, 750</td>
<td>35, 130, 180</td>
<td></td>
</tr>
<tr>
<td>Creep</td>
<td>10</td>
<td>2000, 2500</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>1000, 1500</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>500, 750</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant strain rate test</td>
<td>10</td>
<td></td>
<td></td>
<td>0.005, 0.005, 0.00005</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td></td>
<td></td>
<td>0.005, 0.005, 0.00005</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td></td>
<td></td>
<td>0.005, 0.005</td>
</tr>
<tr>
<td><strong>Tension</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Creep</td>
<td>10</td>
<td>500, 1000, 1500</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>500, 700</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>35</td>
<td>100, 150</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant strain rate test</td>
<td>20</td>
<td></td>
<td></td>
<td>0.0167, 0.00167</td>
</tr>
</tbody>
</table>
Outline

Model Validation
1- Model can predict creep-recovery data at different temperatures and stress levels. (Compression)

\[ T = 10^\circ C \]
\[ \sigma = 2000 \text{kPa} \]

\[ T = 10^\circ C \]
\[ \sigma = 2500 \text{kPa} \]

Creep-recovery test
Compression
@ T=10\textdegree C
Model Validation (Creep-Recovery Test)

1- Model can predict creep-recovery data at different temperatures and stress levels. (Compression)

\[ T = 20^\circ C \]
\[ \sigma = 1000\text{kPa} \]

\[ T = 20^\circ C \]
\[ \sigma = 1500\text{kPa} \]
Model Validation (Creep-Recovery Test)

1- Model can predict creep-recovery data at different temperatures and stress levels. (Compression)

\[ T = 40^\circ C \]
\[ \sigma = 500\text{kPa} \]

\[ T = 40^\circ C \]
\[ \sigma = 750\text{kPa} \]
Model Validation (Creep Test)

2-Model predictions agrees well with creep data at different temperatures and stress levels. Tertiary creep behavior is also captured.

\[
T = 10^\circ C \quad \text{and} \quad T = 20^\circ C
\]
Model Validation (Creep Test)

2-Model predictions agrees well with creep data at different temperatures and stress levels. Tertiary creep behavior is also captured.

$T = 40^\circ C$
Model Validation (Constant Strain Rate Test)

3-Model predicts temperature and rate-dependent behavior of asphalt mixes. Post peak behavior is captured well.

Constant strain rate test
Compression
Loading rate: 0.005/Sec
Model Validation (Constant Strain Rate Test)

3-Model predicts temperature and rate-dependent behavior of asphalt mixes. Post peak behavior is captured well.

Constant strain rate test
Compression
Loading rate: 0.0005/Sec

Constant strain rate test
Compression
Loading rate: 0.00005/Sec
Model Validation in Tension (Creep Test)

4-Model predicts experimental data in tension. Tertiary stage and time of failure are captured well.

Creep test
Tension
Temperature: 10°C

Creep test
Tension
Temperature: 20°C
Model Validation in Tension (Creep Test)

4-Model predicts experimental data in tension. Tertiary stage and time of failure are captured well.

Creep test
Tension
Temperature: 35°C
Model Validation in Tension (Creep Test)

4-Model predicts experimental data in tension.

Constant strain rate test
Tension
Temperature: 20°C
Outline

Implementation procedure
Implementation Procedure

**Procedure to Run the Performance Prediction Continuum Damage Model**

**Fortran**
Fortran compiler is used to compile UMAT (i.e. translates programming commands into action).

**UMAT**
A Fortran code includes the Continuum Damage Model

Abaqus interface

Creating geometry, mesh, and applying loading

Mesh
Boundary Conditions

Pavement Section
Loaded region

Abaqus calls UMAT to run the Continuum Damage Model and UMAT gives Abaqus the material response

Simulating Rutting

Running and viewing the simulation results
Application of the model for rutting performance simulation
Application for Simulation of Wheel Tracking Test

2D FE Model

3D FE Model
## Application: Different Loading Cases

<table>
<thead>
<tr>
<th>Mode</th>
<th>Loading approach</th>
<th>Loading Area</th>
<th>Schematic representation of loading modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (2D)</td>
<td>Pulse loading (plane strain)</td>
<td>One wheel</td>
<td><img src="image1.png" alt="Schematic" /></td>
</tr>
<tr>
<td>2 (2D)</td>
<td>Equivalent loading (plane strain)</td>
<td>One wheel</td>
<td><img src="image2.png" alt="Schematic" /></td>
</tr>
<tr>
<td>3 (2D)</td>
<td>Pulse loading (axisymmetric)</td>
<td>One wheel</td>
<td><img src="image3.png" alt="Schematic" /></td>
</tr>
<tr>
<td>4 (2D)</td>
<td>Equivalent loading (axisymmetric)</td>
<td>One wheel</td>
<td><img src="image4.png" alt="Schematic" /></td>
</tr>
<tr>
<td>5 (3D)</td>
<td>Pulse loading</td>
<td>One wheel</td>
<td><img src="image5.png" alt="Schematic" /></td>
</tr>
<tr>
<td>6 (3D)</td>
<td>Equivalent loading</td>
<td>One wheel</td>
<td><img src="image6.png" alt="Schematic" /></td>
</tr>
<tr>
<td>7 (3D)</td>
<td>Pulse loading</td>
<td>Whole wheel path</td>
<td><img src="image7.png" alt="Schematic" /></td>
</tr>
<tr>
<td>8 (3D)</td>
<td>Equivalent loading</td>
<td>Whole wheel path</td>
<td><img src="image8.png" alt="Schematic" /></td>
</tr>
<tr>
<td>9 (3D)</td>
<td>Pulse loading</td>
<td>Circular loading area</td>
<td><img src="image9.png" alt="Schematic" /></td>
</tr>
<tr>
<td>10 (3D)</td>
<td>Equivalent loading</td>
<td>Circular loading area</td>
<td><img src="image10.png" alt="Schematic" /></td>
</tr>
<tr>
<td>11 (3D)</td>
<td>Moving loading</td>
<td>One wheel</td>
<td><img src="image11.png" alt="Schematic" /></td>
</tr>
</tbody>
</table>
Application: Different Loading Modes and Constitutive Models

2D Rutting simulation results: Different constitutive models.
Application: Different Loading Modes and Constitutive Models

3D Rutting simulation results: Different constitutive models.

Viscoelastic-Viscoplastic Constitutive Model

Elastic-Viscoplastic Constitutive Model
Simplified loading assumptions can be used when using elasto-viscoplastic model.

Simplified loading assumptions should be used carefully when viscoelastic response is significant.

Using simplified loading assumptions causes significant error when the damage level is significant.
Simulation Results

Viscoplastic strain

2D Results

3D Results
Simulation Results

Damage evolution

2D Results

3D Results
Compare with Experimental Data

![Graph showing Rutting (mm) vs. Cycles (N) with lines for Experimental Measurements, 2D results, and 3D results.](image-url)
ALF Data-Variable Stress

Applied stress (T=55\degree C)

First Loading block (The loading time is a constant 0.4 sec following rest time 200 sec)
Model Validation: ALF Data (Strain Response)

1st Cycle

2nd Cycle

3rd Cycle

4th Cycle
Model Validation: ALF Data (Strain Response)

5th Cycle

6th Cycle

7th Cycle

8th Cycle
At large loading cycles model predictions using VE-VP deviates from experimental measurements.

This deviation is due to damage and should be compensated using the damage model.
Model Validation: ALF Data (Strain Response)

Constant Stress-Variable Time Loading

![Graph showing viscoplastic strain over cycle]

- **Cycle** axis ranges from 0 to 60.
- **Viscoplastic Strain** axis ranges from 0 to 0.035.
- The graph compares model predictions (triangle) and experimental data (square) for viscoplastic strain evolution with increasing cycle number.
Conclusions and Future Works

Conclusions:

- Proposed viscoelastic-viscoplastic-viscodamage model predicts rate-, time-, and temperature-dependent behavior of asphalt mixes in both tension and compression.
- Model can be used to predict performance simulations.

Challenges and future works:

- Including healing to the model.
- Including environmental effects such as aging and moisture induced damage to the model.
- Validating the model over an extensive experimental measurements.
- Performing performance simulations in pavements.